INTERPLANETARY HYDROMAGNETIC CLOUDS AS FLARE-GENERATED SPHEROMAKS

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Abstract. Solar flare-generated interplanetary clouds are proposed to be treated as oblate spheromaks (oblamaks) with predominantly force-free magnetic field. The solution found for a force-free field equation in spheroidal coordinates makes it possible to describe the spheromak magnetic fields by a series of spheroidal wave functions. Comparison between theoretical and experimental results is shown in the case of the hydromagnetic cloud from the November 22, 1977 flare (STIP Interval IV).

1. Introduction

The elements of the theory for flare-generated hydromagnetic clouds (Parker, 1957; Chandrasekhar and Kendall, 1957; Freire, 1966) were constructed soon after Morrison (1956) set forth the hypothesis that such clouds may give rise to cosmic ray Forbusch-decreases. Recently, the same theory has been developed as applied to the spheroidal configurations of laboratory plasma with poloidal and toroidal magnetic fields, so called spheromaks (Rosenbluth and Bussac, 1979; Ag *et al.*, 1982). The present-day state of solar-terrestrial physics makes it desirable that the theory should be developed further as applied to flare-generated coronal and interplanetary clouds (Dryer, 1982; Ivanov and Harshiladze, 1984a, b; Pneuman and Cargill, 1984). For brevity, such clouds will henceforth be sometimes called interplanetary spheromaks. One of the tasks of the theory is to construct a mathematical formalism for describing the magnetic field of the clouds. The known spherically-symmetric solutions (Chandrasekhar and Kendell, 1957; Rosenbluth and Bussac, 1979) are hardly applicable because the clouds seem to be strongly oblate (Ivanov, 1982; Ivanov *et al.*, 1982; Watanabe and Kakinuma, 1984). Given below will be the solution for the equation of a force-free magnetic field in the coordinates of an oblate ellipsoid and the application of the solution to a description of the magnetic cloud from the November 22, 1977 flare.

2. Input Equations

We proceed from the force-free field equation (Chandrasekhar and Kendell, 1957; Rosenbluth and Bussac, 1979)

$$\mathbf{j} = k\mathbf{B}, \tag{1}$$

where k is a constant; $\mathbf{B}$ is given by

$$\mathbf{B} = \mathbf{r} \times \nabla\psi + \frac{1}{k}\,\nabla \times (\mathbf{r} \times \nabla\psi) \tag{2}$$

with **r** being radius vector and the function ψ satisfying the Helmholtz equation

$$\nabla^2\psi + k^2\psi = 0\,. \tag{3}$$

Each term in (2) is a solution for the Helmholtz equation of the field **B**,

$$\nabla^2\mathbf{B} + k^2\mathbf{B} = 0\,, \tag{4}$$

but only the complete expression (2) is the solution for both the equation (1) and its consequence, Equation (4). In the coordinates of an oblate ellipsoid of revolution $0 \le \eta \le \infty$, $-1 \le \xi \le 1$, $0 \le \varphi \le 2\pi$, which are related to the Cartesian coordinates

$$\begin{aligned} x &= c_2(1 - \xi^2)^{1/2}(1 + \eta^2)^{1/2}\cos\varphi\,,\\ y &= c_2(1 - \xi^2)^{1/2}(1 + \eta^2)^{1/2}\sin\varphi\,,\\ z &= c_2\xi\eta\,, \end{aligned} \tag{5}$$

where $c_2 = (b^2 - a^2)^{1/2}$, a, b are the ellipsoid semiaxes, the variables are separated and the solution is presented by the spheroidal wave functions (Morse and Feshbach, 1953; Flammer, 1957)

$$\begin{aligned} \psi &= \sum a_{mn}\psi_{mn}\,,\\ \psi_{mn} &= S_{mn}(-ic, \xi)R_{mn}(-ic, i\eta)e^{im\varphi}, \end{aligned} \tag{6}$$

satisfying the equations

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}\xi}\left[(1 - \xi^2)\,\frac{\mathrm{d}S_{mn}}{\mathrm{d}\xi}\right] + \left[\lambda_{mn} + c^2\xi - \frac{m^2}{1 - \xi^2}\right]S_{mn} = 0\,,\\ &\frac{\mathrm{d}}{\mathrm{d}\eta}\left[(1 + \eta^2)\,\frac{\mathrm{d}R_{mn}}{\mathrm{d}\eta}\right] - \left[\lambda_{mn} - c^2\eta + \frac{m^2}{1 + \eta^2}\right]R_{mn} = 0\,, \end{aligned} \tag{7}$$

where m, λ_{mn} are the separation constants; $c = kc_2$. The solutions for (7) are of the form

$$S_{mn} = \sum_{r=0,\,1}^{\infty}{}' \; d_r^{mn}(ic)P_{m+r}^m(\xi)\,, \tag{8}$$

$$\begin{aligned} R_{mn} = {} & \frac{1}{\displaystyle\sum_{r=0}^{\infty}{}' \, d_r^{mn}\,\frac{(2m + r)!}{r!}}\left(\frac{1 + \eta^2}{\eta^2}\right)^{m/2}\sum_{i=0,\,1}^{\infty}{}' \; i^{r+m-n}\,d_r^{mn}\,\times\\ & \times\frac{(2m + r)!}{r!}\,j_{m+r}(c\eta)\,, \end{aligned}$$

where P_{m+r}^m are the associated Legendre functions; d_r^{mn} are the expansion coefficients tabulated by Flammer (1957);

$$j_{m+r}(c\eta) = (c\eta)^{m+r}\left(-\frac{1}{c\eta}\,\frac{\mathrm{d}}{\mathrm{d}(c\eta)}\right)^{m+r}\frac{\sin c\eta}{c\eta} \tag{9}$$

is the spherical Bessel function; Σ' denotes the summation of only even terms over r when $(n - m)$ is even, and of only odd terms when $(n - m)$ is odd.

3. General Solution

In the coordinates ξ, η, φ, the relation (2) takes the form

$$B_\eta = \frac{c_2^2}{kh_\xi h_\varphi}\left[\frac{h_\varphi}{h_\eta h_\xi}\left(\eta \frac{\partial^2\psi}{\partial\xi^2} + \frac{\partial\psi}{\partial\eta} + \xi \frac{\partial^2\psi}{\partial\eta\,\partial\xi}\right) - \right.$$

$$\left. - \frac{2c_2^3\xi}{h_\eta^4}\left(\eta \frac{\partial\psi}{\partial\xi} + \xi \frac{\partial\psi}{\partial\eta}\right) + \frac{h_\xi\eta}{h_\eta h_\varphi}\frac{\partial^2\psi}{\partial\varphi^2} - k\xi \frac{\partial\psi}{\partial\varphi}\right],$$

$$B_\xi = \frac{c_2^2}{kh_\eta h_\varphi}\left[\frac{2c_2^3\eta}{h_\xi^4}\left(\eta \frac{\partial\psi}{\partial\xi} + \xi \frac{\partial\psi}{\partial\eta}\right) - \frac{h_\varphi}{h_\eta h_\xi}\left(\xi \frac{\partial^2\psi}{\partial\eta^2} + \right.\right.$$

$$\left.\left. + \eta \frac{\partial^2\psi}{\partial\eta\,\partial\xi} + \frac{\partial\psi}{\partial\xi}\right) - \frac{h_\eta\xi}{h_\xi h_\varphi}\frac{\partial^2\psi}{\partial\varphi^2} - k\eta \frac{\partial\psi}{\partial\varphi}\right],$$

$$B_\varphi = \frac{c_2^2}{kh_\eta h_\xi}\left[k\left(\eta \frac{\partial\psi}{\partial\xi} + \xi \frac{\partial\psi}{\partial\eta}\right) + \frac{h_\eta}{h_\xi h_\varphi}\left(\xi \frac{\partial^2\psi}{\partial\xi\,\partial\varphi} + \right.\right.$$

$$\left.\left. + \frac{\partial\psi}{\partial\varphi}\right) - \frac{h_\xi}{h_\eta h_\varphi}\left(\eta \frac{\partial^2\psi}{\partial\eta\,\partial\varphi} + \frac{\partial\psi}{\partial\varphi}\right)\right],$$

$$h_\eta = c_2\left(\frac{\eta^2 + \xi^2}{1 - \eta^2}\right)^{1/2}, \qquad h_\xi = c_2\left(\frac{\eta^2 + \xi^2}{1 - \xi^2}\right)^{1/2},$$

$$h_\varphi = c_2(1 - \xi^2)^{1/2}(1 + \eta^2)^{1/2}. \tag{10}$$

Considering (6)–(9), the formulae (10) are the general solution of Equation (4) for a force-free field at k = const. in the coordinates of an oblate spheroid. Of course, they may be imparted in any convenient form. We shall not write the tedious detailed solution for **B**.

The solution in the coordinates of a prolate spheroid is obtainable from (6), (8)–(10) by the conventional transformation $\eta \to -i\eta$, $c \to -ic$.

4. Axially-Symmetric Solutions

Examine the axially-symmetric solutions $\partial\psi/\partial\varphi = 0$, $m = 0$:

$$B_\eta = \frac{(1 - \xi^2)(1 + \eta^2)^{1/2}}{(\eta^2 + \xi^2)^{3/2}}\left(\eta \frac{\partial^2\psi}{\partial\xi^2} + \xi \frac{\partial^2\psi}{\partial\eta\,\partial\xi} + \frac{\partial\psi}{\partial\eta}\right) - \frac{2\xi(1 + \eta^2)^{3/2}}{(\eta^2 + \xi^2)^{5/2}}\left(\eta \frac{\partial\psi}{\partial\xi} + \xi \frac{\psi}{\partial\eta}\right),$$

$$B_\xi = -\frac{(1+\eta^2)(1-\xi^2)^{1/2}}{(\eta^2+\xi^2)^{3/2}}\left(\xi\frac{\partial^2\psi}{\partial\eta^2} + \eta\frac{\partial^2\psi}{\partial\eta\,\partial\xi} + \frac{\partial\psi}{\partial\xi}\right) +$$

$$+ 2\eta\frac{(1-\xi^2)^{3/2}}{(\eta^2+\xi^2)^{5/2}}\left(\eta\frac{\partial\psi}{\partial\xi} + \xi\frac{\partial\psi}{\partial\eta}\right),$$

$$B_\varphi = \frac{(1+\eta^2)^{1/2}(1-\xi^2)^{1/2}}{(\eta^2+\xi^2)}\left(\eta\frac{\partial\psi}{\partial\xi} + \xi\frac{\partial\psi}{\partial\eta}\right). \tag{11}$$

The coefficients d_r^{0n} are rapidly decreasing; therefore, we may take for numerical estimations:

$$\psi \simeq a_{00}\psi_{00} + a_{01}\psi_{01} = a_{00}P_0^0(\xi)j_0(c\eta) + a_{01}P_1^0(\xi)j_1(c\eta).$$

On substituting ψ in (11), we get:

$$B_{00\eta} = \frac{a_{00}d_0^{00}(1+\eta^2)(\eta^2-\xi^2-\xi^4-3\eta^2\xi^2)(c\eta\cos c\eta - \sin c\eta)}{c\eta^2(\eta^2+\xi^2)^{5/2}},$$

$$B_{00\xi} = \frac{a_{00}d_0^{00}\xi(1-\xi^2)^{1/2}}{c\eta^3(\eta^2+\xi^2)^{5/2}}\,[2\eta^2(1-\xi^2)(c\eta\cos c\eta - \sin c\eta) -$$

$$- (1+\eta^2)(\eta^2+\xi^2)(2\sin c\eta - c^2\eta^2\sin c\eta - 2c\eta\cos c\eta)],$$

$$B_{00\varphi} = \frac{a_{00}d_0^{00}\xi(1+\eta^2)^{1/2}(1-\xi^2)^{1/2}(\eta c\cos c\eta - \sin c\eta)}{c\eta^2(\eta^2+\xi^2)},$$

$$B_{01\eta} = \{2\xi(1+\eta^2)^{1/2}[(a_{01}d_0^{01})(1-\xi^2)^{3/2}(2c\eta\cos c\eta - 2\sin c\eta + c^2\eta^2\sin c\eta) -$$

$$- \eta^3(1+\eta^2)^{1/2}c^2B_{01\varphi}]\}\,/\,[c^2\eta^3(\eta^2+\xi^2)^{3/2}(1-\xi^2)^{1/2}],$$

$$B_{01\xi} = \frac{(1-\xi^2)^{1/2}}{c^2\eta^4(1+\eta^2)(\eta^2+\xi^2)^{3/2}}\,\{2\eta^5c^2(1-\xi^2)^{1/2}(1+\eta^2)^{1/2}B_{01\varphi} -$$

$$- (a_{01}d_1^{01})(1+\eta^2)^2[c\eta^3\cos c\eta - \eta^2\sin c\eta + c^2\eta^4\sin c\eta +$$

$$+ \xi(c^3\eta^3\cos c\eta - 6c\eta\cos c\eta + 6\sin c\eta - 3c^2\eta^2\sin c\eta)]\},$$

$$B_{01\varphi} = \frac{a_{01}d_1^{01}(1+\eta^2)^{1/2}(1-\xi^2)^{1/2}}{\eta^3(\eta^2+\xi^2)c^2}\,\{\xi^2[2\eta c\cos c\eta - (2-c^2\eta^2)\sin c\eta] +$$

$$+ \eta^2(\sin c\eta - \eta c\cos c\eta)\}. \tag{12}$$

The transition to the Cartesian coordinates is made using the formulae

$$B_x = -\xi\left(\frac{1+\eta^2}{\eta^2+\xi^2}\right)^{1/2}B_\xi\cos\varphi + \eta\left(\frac{1-\xi^2}{\eta^2+\xi^2}\right)^{1/2}B_\eta\cos\varphi - B_\varphi\sin\varphi,$$

$$B_y = -\xi\left(\frac{1+\eta^2}{\eta^2+\xi^2}\right)^{1/2}B_\xi\sin\varphi + \eta\left(\frac{1-\xi^2}{\eta^2+\xi^2}\right)^{1/2}B_\eta\sin\varphi + B_\varphi\cos\varphi,$$

$$B_z = \eta\left(\frac{1-\xi^2}{\eta^2+\xi^2}\right)^{1/2} B_\xi + \xi\left(\frac{1+\eta^2}{\eta^2+\xi^2}\right)^{1/2} B_\eta \,. \tag{13}$$

Figure 1 shows an example of the magnetic field components $\mathbf{B}_{00}$ and $\mathbf{B}_{01}$ calculated from (12), (13) at $x/c_2 = -0.8$, $-2 \leqslant y/c_2 \leqslant 2$, $z/c_2 = 0.4$, $c = 1$, $a_{00} = a_{01} = -1$.

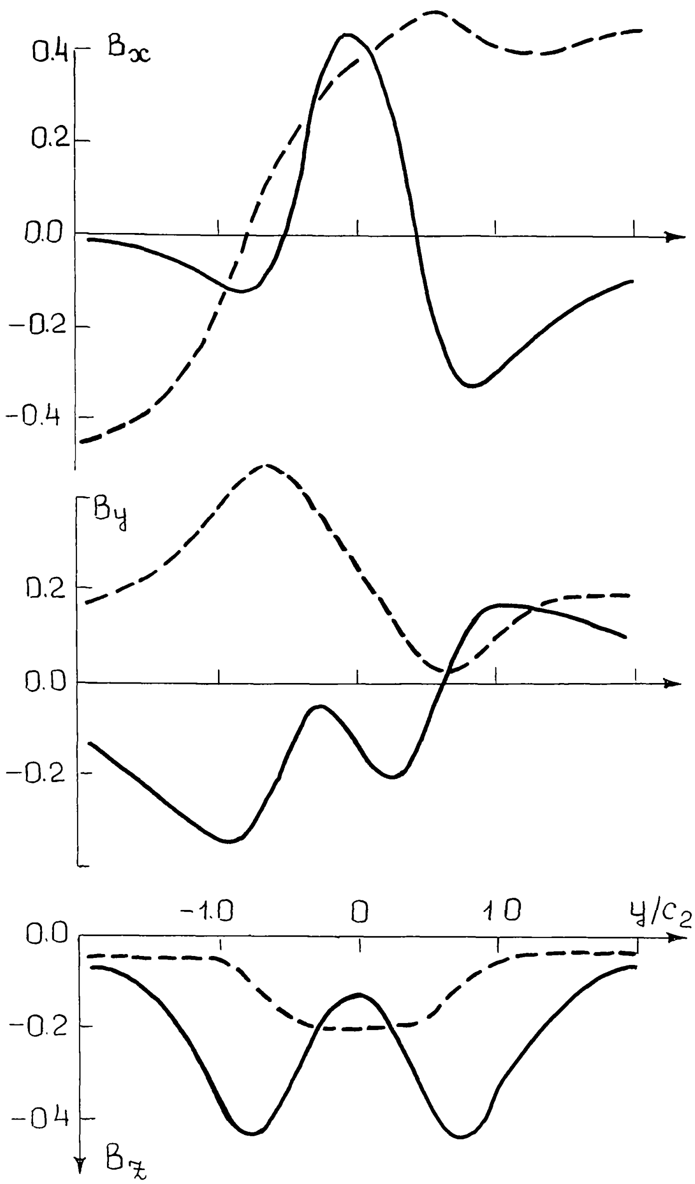

Fig. 1. Magnetic fields corresponding to spheroidal wave functions ψ_{00} and ψ_{01} (dotted line) in relative units on the section $x/c_2 = -0.8$, $z/c_2 = 0.4$, and $-2 \leqslant y/c_2 \leqslant 2$.

5. Interaction with Interplanetary Medium

In the classical spheromak (Chandrasekhar and Kendell, 1957; Rosenbluth and Bussak, 1979), the particular solutions for Equation (3),

$$B_r = n(n+1)P_n^m \frac{j_n(kr)}{kr} e^{im\varphi}$$

(but not their combination) at such values of $r = r_0$ that $j_n(kr) = 0$, $B_r = 0$, make sure that spherical spheromaks with their field confined within a sphere of radius r_0 will arise. We could not find any analogous ellipsoidal spheromaks in (6)–(10). It should be noted that the solution for Equation (1) in which $B_r = 0$ on the ellipsoid surface was called ellipsoidal spheromak (Rosenbluth and Bussak, 1979). However, the normal component **B**, rather that B_r, should vanish on the ellipsoidal spheromak surface. The interplanetary spheromaks (magnetic clouds) are probably formed in the interactions of flare ejections with the coronal and interplanetary medium when the fields, which are approximately described by (6)–(13), are confined within a magnetic cavity whose boundary $F(\eta, \xi, \varphi) = 0$ satisfies the condition (Beard, 1964)

$$|\mathbf{n}_s \times \mathbf{B}| = -(8\pi x\rho v^2)^{1/2}\, \mathbf{n}_s \mathbf{n}_v\,, \qquad \mathbf{n}_s = \nabla F/|\nabla F|\,,$$

where $x\rho v^2$ is the dynamic head onto a spheromak; $\mathbf{n}_s$ and $\mathbf{n}_v$ are unit vectors of boundary normal and incoming flux velocity.

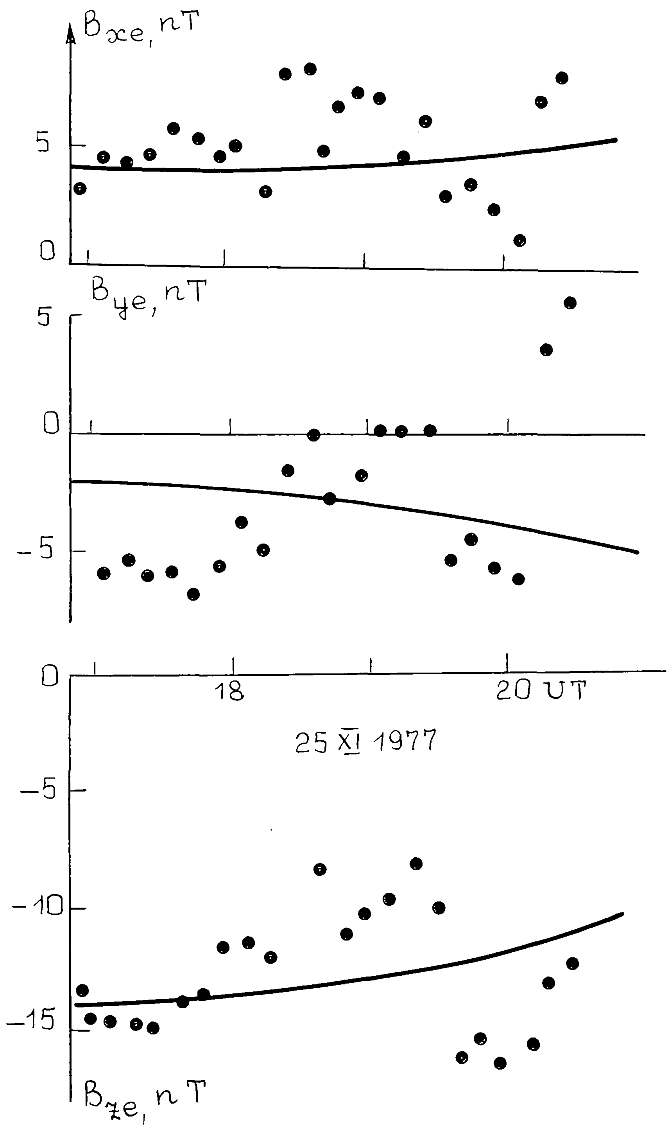

Fig. 2. Magnetic fields of the interplanetary hydromagnetic cloud from the solar flare on November 22, 1977: theoretical curves and experimental points presented in solar ecliptic coordinates.

In principle, the boundary F and the field **B** are obtained, on the analogy of the Earth's magnetosphere, by the standard method, for example by the Beard self-consistent method (Beard, 1964) if one takes the solutions (10)–(12) for **B** as a first approximation.

6. Example

Examine an interplanetary spheromak shaped as an oblate ellipsoid of revolution. Let the ellipsoid center be the origin of the Cartesian coordinate system (5) whose x, y, and z axes are correspondingly) parallel to the z_{se}, x_{se}, and y_{se} of the solar-ecliptic coordinate system. Let the Earth traverse the spheromak along the trajectory $x/c_2 = -0.8$, $z/c_2 = 0.4$, and $y/c_2 \neq$ const. This situation corresponds roughly to the real case when the Earth traversed the hydromagnetic cloud produced by the powerful solar flare of November 22, 1977 (Ivanov and Mikerina, 1983). Therefore, the above described theory may tentatively be compared with experimental data. The formulae (12) have been obtained for

$$\psi \simeq a_{00} \frac{\sin c\eta}{c\eta} - a_{01} \frac{c\eta \cos c\eta - \sin c\eta}{c^2\eta^2} \xi ,$$

and Figure 1 – for $a_{00} - a_{01} = -1$.

Figure 2 shows the theoretical curves ($c = 1, a_{01} = -1.3 \times 10^{-4}, a_{00} = -3.3 \times 10^{-4}$ CGSM, $y/c_2 = (-2.0) - (-1.2)$ and experimental points (ISEE-2 data on November 25, 1977).

The comparison between the theory and the experimental data from ISEE-2 magnetic field components of the IMF reveal the fact that the field distortions due to the spheromak's interaction with the interplanetary medium are not explicitly included in the comparison. This could be done only with fully-MHD, time-dependent computations.

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