# Propagation of Fast Coronal Mass Ejections and Shock Waves Associated with Type II Radio-Burst Emission: An Analytic Study 

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#### Abstract

Coronal mass ejections (CMEs) are large-scale eruptive events in the solar corona. Once they are expelled into the interplanetary (IP) medium, they propagate outwards and "evolve" interacting with the solar wind. Fast CMEs associated with IP shocks are a critical subject for space weather investigations. We present an analytic model to study the heliocentric evolution of fast CME/shock events and their association with type II radio-burst emissions. The propagation model assumes an early stage where the CME acts as a piston driving a shock wave; beyond this point the CME decelerates, tending to match the ambient solar wind speed and its shock decays. We use the shock speed evolution to reproduce type II radio-burst emissions. We analyse four fast CME halo events that were associated with kilometric type II radio bursts, and in-situ measurements of IP shock and CME signatures. The results show good agreement with the dynamic spectra of the type II frequency drifts and the in-situ measurements. This suggests that, in general, IP shocks associated with fast CMEs evolve as blast waves approaching 1 AU , implying that the CMEs do not drive their shocks any further at this heliocentric range.


Keywords Coronal mass ejections, interplanetary • Coronal mass ejections, theory • Radio bursts, type II

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## 1. Introduction

Coronal mass ejections (CMEs) are complex events involving the release of large amounts of material, energy, and magnetic field from the Sun into the solar wind. They may also involve other solar phenomena, such as solar flares, solar energetic particles, solar radio bursts, and interplanetary (IP) shocks (Forsyth et al., 2006; Webb and Gopalswamy, 2006). In general, CMEs propagate through the IP medium with initial velocities between $200-2000 \mathrm{~km} \mathrm{~s}^{-1}$ (Vourlidas et al., 2000). Fast CMEs associated with IP shocks are the main cause of intense geomagnetic storms (e.g. Ontiveros and Gonzalez-Esparza, 2010, and references therein), making the propagation of fast CMEs and their shocks a crucial issue in space weather studies. As pointed out by Vršnak $(2006,2008)$, we need to grasp the physical mechanisms that govern the CME evolution to understand when, where, and how fast CMEs propagate and decelerate in the IP medium.

Recently white-light observations, combining data from LASCO, SMEI, and the Heliospheric Imagers (HIs), tracked a few CMEs from the Sun to 1 AU, helping us to increase our knowledge of the CME kinematics (e.g. Harrison et al., 2008, Webb et al., 2009a, 2009b, Liu et al., 2010a, 2010b). However, in general, it is very difficult to fully track these events, so we need more observations and developments to address this problem. There are several theoretical, numerical, and empirical studies to describe the CME dynamics (Forbes et al., 2006). CMEs interact with the ambient solar wind, and this interaction decelerates (accelerates) the fast (slow) CMEs (Gopalswamy et al., 2000). In general, analytical models focus on the dynamics in the CME-solar wind interaction; such as linear and quadratic drag forces (Cargill, 2004; Vršnak and Gopalswamy, 2002), mass accretion (Tappin, 2006), and viscous and turbulent forces (Borgazzi et al., 2009). All these models assume a direct interaction between the CME and the ambient solar wind without taking into account the role of the driven IP shock and its plasma sheath (shocked solar wind).

It is possible to track fast CME/shock events using electromagnetic wave observations by spacecraft. Solar radio bursts of type II are characterised by a narrow band of intense radiation of which frequency drifts downwards with time and distance from the Sun over time scales from a few hours to one or two days. Type II bursts are produced by the excitation of plasma waves by a shock propagating through the solar wind (Cane and Stone, 1984; Cane, Sheeley, and Howard, 1987). These emissions occur at the fundamental and/or harmonic of the plasma frequency $\left(f_{\mathrm{p}}\right)$ which is related to the square root of the electron plasma density ( $n$ ) at the source region (see Equation (16)). The type II radio bursts are typically observed in the metre-wave regime at frequencies less than 150 MHz . However, type II radio bursts have been observed to start at frequencies as high as 500 MHz (Nakajima et al., 1990; Vršnak et al., 1995). It is well established that decametric/hectometric (DH) to kilometric $(\mathrm{km})$ type II radio emissions are caused by the propagation of fast CME/shocks through the interplanetary medium (Cane, Sheeley, and Howard, 1987). However, not necessarily all the CME/shock events generate type II emissions (Gopalswamy et al., 1998, 2008). The type II frequency-drifting can, in principle, provide continuous tracking of some CME/shock events from the solar corona through the heliosphere. Therefore, these radio observations can be used to approximate the speed profile of a CME/shock (Reiner, Kaiser, and Bougeret, 2007). Furthermore, Gonzalez-Esparza and Aguilar-Rodriguez (2009) used the type II frequency drifts to calculate shock speeds, at some convenient intervals, tracking the shock deceleration in the IP medium; Lara and Borgazzi (2009) calculated the synthetic type II radio burst associated with the analytic trajectory of a CME using an analytic model.

Empirical studies on the propagation of IP shocks and fast CMEs point out relationships between shock arrival speeds, shock transit times, CME properties, and solar wind characteristics (e.g. Gopalswamy et al., 2005; Kim, Moon, and Cho, 2007). However, when they
are tested against several case events, the empirical models obtain large uncertainties, suggesting that we require a different perspective to address this problem (Cho et al., 2003; Kim, Moon, and Cho, 2007).

Dryer (1974) studied blast shocks associated with solar flares from analytical and numeric perspectives. Smart and Shea (1985) also investigated IP shocks related to solar flares, they found that IP shocks propagation presents two stages:
i) a short period of constant speed where the shock is driven by a piston for which speed and duration were derived from metric type II bursts and solar flare intensity, and
ii) a blast wave decaying evolution.

Pinter and Dryer (1990) applied these results to calculate transit times (TT) to 1 AU of some shocks associated with solar flares, obtaining good agreement with observations. They also found that the characteristics of the driving stage affect the shock TT and arrival speed. On the other hand, there are reports of a similar heliocentric evolution in shocks associated with fast CMEs. Combining coronagraph, interplanetary scintillation and in-situ measurements, Manoharan et al. (2001), Manoharan (2006, 2010) and Pohjolainen et al. (2007) tracked the propagation of some fast CME/shock events, suggesting that they also present two propagation stages within 1 AU : an initial one, near the Sun, with a small deceleration up to a certain heliocentric distance, beyond which there is a large deceleration where the CME/shock tends to equal the ambient wind speed.

These two propagation stages in the heliocentric evolution of fast CME/shock events are also found in numerical and analytical models. González-Esparza et al. (2003a, 2003b, 2007) developed 1D hydrodynamic (HD) numerical simulations of fast CMEs in the IP medium. They found that a fast CME front and its shock present an initial quasi-constant speed propagation followed by a decaying speed at farther heliocentric distances. On the other hand, Cantó et al. (2005) presented a full analytic HD model to calculate CME trajectories and TTs. This model also finds the two propagation stages. Corona-Romero and Gonzalez-Esparza (2011) investigated the similarities between the numerical and analytical models commented on before. They concluded that the evolution of a fast CME/shock is described by three dynamic phases: driving, decoupling, and decaying. In the driving phase, the fast CME propagates with a quasi-constant speed, a condition which allows it to transfer momentum forward, driving the shock. On the other hand, in the final decaying stage, the fast CME decelerates, tending to equal the ambient wind speed and its shock evolves as a blast wave slowing to become a magnetosonic perturbation. The intermediate decoupling phase bonds these two opposite dynamic states, implying a ceasing transfer of momentum through the plasma sheath, from the CME front to the shock.

We study the propagation of fast CME/shock events, to compare the evolution of the shock speed with type II radio burst drifts and in-situ measurements. In order to do so, we start from a piston-shock analytical model to describe the CME and shock evolutions. We modify this model to adapt it to more realistic scenarios (Section 2). In Section 3 we apply the model to analyse four study cases, calculating the trajectories of CMEs, IP shocks, and simulating the type II radio-burst emission associated with the shock propagation. Finally, we present our summary and conclusions in Section 4.

## 2. Analytical Model of Fast CME/Shock Propagation

Cantó, Raga, and D’Alessio (2000) developed an analytical formalism to describe the dynamics of two interacting hypersonic fluids; such a formalism was based on the conservation


Figure $1 \mathrm{CME} /$ shock model for times (a) $t<\tau_{c 1}$ and (b) $t>\tau_{c 2}$. The croissant-like shape represents the fast CME with radius $R_{\text {cme }}$ and the shock is propagating ahead. The dotted thin lines represent the magnetic field lines. $r_{\mathrm{cme}}$ and $r_{\mathrm{sh}}$ are the heliocentric positions of the CME leading edge and the shock, respectively. The separation between the CME front and the shock is the standoff distance ( $d_{\mathrm{so}}$ ). (a) During the driving stage $\left(t<\tau_{c 1}\right)$ the CME and shock wave have the same speed and $d_{\text {so }}$ is almost constant. (b) In the decaying stage $\left(t>\tau_{c 2}\right)$ the CME decelerates, tending to equal the solar wind speed, and the shock wave decays to become a magnetosonic perturbation, $d_{\mathrm{so}}$ increases and the plasma sheath expands and relaxes into normal solar wind.
of linear momentum. Subsequently, Cantó et al. (2005) adapted this formalism to describe the propagation of supersonic CMEs through the solar wind (SW). They considered the CME as a dense and fast fluid injected into the ambient wind. The SW opposition to the CME propagation derives into the CME deceleration by the equilibrium of linear momentum between the CME and ambient wind. The CME is injected at an initial position (beyond the critical point) during the injection time. In the present work we use the notation $\Delta t_{f}$ and $r_{0}$ for the injection time and the initial position, respectively.

Based on the previously discussed work, Corona-Romero and Gonzalez-Esparza (2011) developed a piston-shock analytical HD model to describe the evolution of the CME driver and its shock in the inner heliosphere. They compared their model with 1D HD numerical simulations of CME/shock events. In the analytical model they assumed the pristine fast CME as a piston driving a shock wave, and they obtained analytical expressions for the shock propagation. They found good agreement between the model and simulations. The two models show that initially the fast CME propagates at about a constant speed and drives the shock (driving stage) until it reaches a certain distance after which it decelerates and decouples from the shock (decoupling process). Then the CME and its shock decelerate (decaying stage). They also applied the piston shock model to analyse a few fast CME/shock events finding quantitative agreement with CME in-situ data at 1 AU ; however, the shock arrival speeds and their TTs presented some discrepancies with the in-situ observations. They concluded that these inconsistencies could be explained by neglecting the tangential flows inside the plasma sheath. This 1D condition overestimates the plasma-sheath width and the shock speed, thereby causing shorter shock TTs.

Now we extend the model starting from a driver bow shock initial configuration for the fast CME/shock (Ontiveros and Vourlidas, 2009; Vourlidas and Ontiveros, 2009; Maloney and Gallagher, 2011). Figure 1a shows a sketch of the model, the croissant-like CME and the shock wave. The shadow region between the CME leading edge ( $r_{\mathrm{cme}}$ ) and the shock $\left(r_{\mathrm{sh}}\right)$ is the plasma sheath, formed by compressed solar wind. Such geometry considers the
tangential mass fluxes inside the plasma sheath, since the sheath material flows around the CME, solving the problems commented on in the previous paragraph.

We consider the solar wind as a spherically homogeneous polytropic plasma (with polytropic index $\gamma$ ) expanding at a constant rate. Frozen in the solar wind there is a magnetic field initially radial at two solar radii ( $R_{\odot}$ ), and the field source rotates with a frequency $\omega_{\odot}=2.7 \times 10^{-6} \mathrm{~Hz}$ (solar sidereal period at the equator). Figure 1 shows the magnetic field configuration (dotted thin lines); in panel (a) the magnetic field is radial near the Sun, while the field lines are curved at larger heliocentric distances (panel (b)).

### 2.1. CME Propagation

Following Corona-Romero and Gonzalez-Esparza (2011), at the beginning, during the driving stage, the CME leading edge maintains a constant speed. The first critical time ( $\tau_{c 1}$ ) indicates the time when the interaction with the plasma sheath becomes dominant, causing the deceleration of the CME. Thus $\tau_{c 1}$ splits the evolution of the CME speed ( $v_{\text {cme }}$ ):

$$
v_{\mathrm{cme}}(t)= \begin{cases}v_{0 \mathrm{cme}}, & t<\tau_{c 1},  \tag{1}\\ v_{\mathrm{lAU}}\left(1+\frac{(a-1) \sqrt{a c} \Delta t_{f}}{\sqrt{2(a-1) \Delta t_{f} t-a(1-c)\left(\Delta t_{f}\right)^{2}}}\right), & t \geq \tau_{c 1},\end{cases}
$$

where $v_{0 c m e}$ is the speed near the Sun (linear fit of plane-of-sky speed measured by coronagraph images), $v_{1 \mathrm{AU}}$ is the solar wind speed at 1 AU , and $\Delta t_{f}$ the duration of the rise phase associated with the flare. In addition, $\tau_{c 1}$ is defined by

$$
\begin{equation*}
\tau_{c 1}=\frac{a(1+\sqrt{c})}{a-1} \Delta t_{f}, \tag{2}
\end{equation*}
$$

where $a$ and $c$ are related with the CME kinetic properties. $a$ can be expressed by

$$
\begin{equation*}
a=\frac{v_{0 \mathrm{cme}}}{v_{\mathrm{IAU}}}\left(\frac{1+\sqrt{c}}{\sqrt{c}}\right)-\frac{1}{\sqrt{c}}, \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
c=\frac{n_{\text {0cme }}}{n_{1 \mathrm{AU}}}\left(\frac{r_{0 \mathrm{cme}}}{1 \mathrm{AU}}\right)^{2} . \tag{4}
\end{equation*}
$$

$c$ is the ratio between the CME ( $n_{0 \mathrm{cme}}$ ) and the local solar wind ( $n_{1 \mathrm{AU}}\left[1 \mathrm{AU} / r_{0 \mathrm{cme}}\right]^{2}$ ) densities at $t=0$; here $r_{0 \mathrm{cme}}$ and $n_{1 \mathrm{AU}}$ are the initial position of the CME leading edge and the solar wind density at 1 AU , respectively.

The product $a c n_{1 \mathrm{AU}} v_{1 \mathrm{AU}}$ represents the flux of injected CME material during the interval $\Delta t_{f}$. The constants $a$ and $c$ are related with the CME initial inertia and kinetic energy, and they are also related with the ambient SW characteristics at the inner boundary (Equations (3) and (4)). For these reasons, the CME dynamics depends on the values of $a, c$ and $\Delta t_{f}$, which define the duration of driving and decaying phases (Equation (2)) as well the CME deceleration (see Equation (1)).

Finally, the CME leading edge position ( $r_{\text {cme }}$ ) is obtained by integrating Equation (1):

$$
r_{\mathrm{cme}}(t)= \begin{cases}r_{0 \mathrm{cme}}+v_{0 \mathrm{cme}} t, & t<\tau_{c 1}  \tag{5}\\ r_{0 \mathrm{cme}}+v_{1 \mathrm{AU}}\left(t-a c \Delta t_{f}\right) & \\ \quad+v_{1 \mathrm{AU}} \sqrt{2(a-1) a c \Delta t_{f} t-a^{2} c(1-c)\left(\Delta t_{f}\right)^{2}}, & t \geq \tau_{c 1}\end{cases}
$$

Further details of this model and its applications can be found in González and Cantó (2002) and González et al. (2006).

### 2.2. Shock Propagation

The shock begins its propagation as a bow shock driven by the CME (see Figure 1a). During the driving stage $\left(t<\tau_{c 1}\right)$ the CME speed ( $v_{\text {cme }}$ ) is constant and the ratio between the standoff distance ( $d_{\mathrm{so}}$ ) and the CME radius ( $R_{\mathrm{cme}}$ ) depends on the magnetosonic Mach number $\left(M_{1}\right)$ and $\gamma$ (Maloney and Gallagher, 2011 and references therein). The shorter $M_{1}$ the larger $d_{\mathrm{so}} / R_{\mathrm{cme}}$. For large values of $M_{1}, d_{\mathrm{so}} / R_{\mathrm{cme}}$ tends to an asymptotic value ( $\sim 0.23$ ) determined by $\gamma$ (Petrinec, 2002). During the driving stage $M_{1}$ increases due to the solar wind expansion and the constant value of $v_{\mathrm{cme}}$; this decreases the value of $d_{\mathrm{so}} / R_{\mathrm{cme}}$. However, the CME expansion may reduce this effect making the value of $d_{\text {so }}$ relatively constant. For simplicity, we assume that $d_{\text {so }}$ is constant during the driving; implying that, for $t<\tau_{c 1}$, the CME leading edge and shock speeds are equal ( $v_{\mathrm{cme}}=v_{\mathrm{sh}}$ ).

For times $t>\tau_{c 1}$ the CME and shock speeds begin to differ due to the CME deceleration, which increases the distance between the CME front and the shock. As this distance grows, the driving from the CME gradually ceases. Then, a time comes when the plasma-sheath relaxation leads to the end of the driving stage. The second critical time ( $\tau_{c 2}$ ) marks the moment when the CME is no longer capable to drive the bow shock, and consequently the shock begins to decouple and evolve into a blast wave. Figure 1b shows the decaying phase ( $t>\tau_{c 2}$ ), where the CME/shock standoff distance is growing. During this phase the shock fades out and the compression of the plasma sheath decreases.

The evolution of the shock speed $\left(v_{\text {sh }}\right)$ is given by

$$
v_{\mathrm{sh}}(t)= \begin{cases}v_{0 \mathrm{cme}}, & t<\tau_{c 2},  \tag{6}\\ \left(v_{0 \mathrm{cme}}-v_{1 \mathrm{AU}}\right)\left(\frac{t}{\tau_{c 2}}\right)^{-1 / 3}+v_{1 \mathrm{AU}}, & t \geq \tau_{c 2} .\end{cases}
$$

During the first stage of Equation (6), the shock is driven by the CME; the second stage ( $t \geq \tau_{c 2}$ ) is the blast wave solution for a $r^{-2}$ density profile (Cavaliere and Messina, 1976). Moreover, $\tau_{c 2}$ is defined as

$$
\begin{equation*}
\tau_{c 2}=\frac{d_{\mathrm{so}}}{\sqrt{c_{A 2}^{2}+c_{S 2}^{2}}}+\tau_{c 1} \tag{7}
\end{equation*}
$$

where $c_{A 2}$ and $c_{S 2}$ are the Alfvénic and sonic speeds in the plasma sheath at $t=\tau_{c 1}$. The values for $c_{A 2}$ and $c_{S 2}$ are calculated as

$$
\begin{align*}
& c_{A 2}^{2}=c_{A 1}^{2}\left[\frac{B_{*}^{2}}{n_{*}}\right],  \tag{8}\\
& c_{S 2}^{2}=c_{S 1}^{2}\left[\frac{p_{*}}{n_{*}}\right], \tag{9}
\end{align*}
$$

where $c_{A 1}$ and $c_{S 1}$ are the Alfvénic and sonic speeds, respectively. The magnetic field ( $B_{*}$ ), number density $\left(n_{*}\right)$ and thermal pressure ( $p_{*}$ ) jumps across the shock are calculated by the Petrinec and Russell (1997) polytropic jump relations (see the Appendix, Equations (20), (19), and (21)).

To calculate $d_{\text {so }}$ in Equation (7), we combine the Bothmer and Schwenn (1998) empirical relation to estimate the CME radius ( $R_{\text {cme }}$ ) and the Farris and Russell (1994) model to approach the standoff distance. After some manipulation, we obtain a relation between $r_{\mathrm{cme}}$ and $d_{\mathrm{so}}$ :

$$
\begin{equation*}
\frac{d_{\mathrm{so}}}{1 \mathrm{AU}}=0.264\left[\frac{(\gamma-1) M_{1}^{2}+2}{(\gamma+1)\left(M_{1}^{2}-1\right)}\right]\left(\frac{r_{1 \mathrm{cme}}}{1 \mathrm{AU}}\right)^{0.78} \tag{10}
\end{equation*}
$$

where $r_{1 \mathrm{cme}}=r_{0 \mathrm{cme}}+v_{0 \mathrm{cme}} \tau_{c 1}$ is the CME leading edge position at $t=\tau_{c 1} . M_{1}$ is defined by

$$
\begin{equation*}
M_{1}^{2}=\frac{2\left(v_{0 \mathrm{cme}}-v_{1 \mathrm{AU}}\right)^{2}}{c_{A 1}^{2}+c_{S 1}^{2}+\sqrt{\left(c_{A 1}^{2}+c_{S 1}^{2}\right)^{2}-4 \cos ^{2}\left(\theta_{B v}\right) c_{A 1}^{2} c_{S 1}^{2}}} \tag{11}
\end{equation*}
$$

with $\theta_{B v}$ the angle between the IP magnetic field ( $\mathbf{B}_{\mathrm{sw}}$ ) and the shock normal.
The values of $c_{A 1}, c_{S 1}$, and $\theta_{B v}$ are calculated by

$$
\begin{align*}
c_{A 1}^{2} & =\frac{B_{1 \mathrm{AU}}^{2}}{\mu_{0} n_{1 \mathrm{AU}}}\left[\frac{1 \mathrm{AU}}{r_{1 \mathrm{cme}}}\right]^{2}\left[\frac{v_{1 \mathrm{AU}}+\left(r_{1 \mathrm{cme}}-2 R_{\odot}\right) \omega_{\odot}}{v_{1 \mathrm{AU}}+\left(1 \mathrm{AU}-2 R_{\odot}\right) \omega_{\odot}}\right],  \tag{12}\\
c_{S 1}^{2} & =\gamma \frac{2 k_{B} T_{1 \mathrm{AU}}}{m_{p}}\left[\frac{1 \mathrm{AU}}{r_{1 \mathrm{cme}}}\right]^{2(\gamma-1)},  \tag{13}\\
\cos ^{2}\left(\theta_{B v}\right) & =\frac{v_{1 \mathrm{AU}}^{2}}{v_{1 \mathrm{AU}}^{2}+\left(r_{1 \mathrm{cme}}-2 R_{\odot}\right)^{2} \omega_{\odot}^{2}}, \tag{14}
\end{align*}
$$

where $n_{1 \mathrm{AU}}, B_{1 \mathrm{AU}}$, and $T_{1 \mathrm{AU}}$ are the values of proton density, magnetic field intensity, and proton temperature at 1 AU , respectively. In Equation (13) we consider an equal number of protons and electrons, and in Equations (12), (13) and (14) we apply the frozen-in Parkerlike magnetic field assumption.

Finally, the shock position $\left(r_{\text {sh }}\right)$ is obtained by integrating Equation (6):

$$
r_{\mathrm{sh}}(t)= \begin{cases}v_{0 \mathrm{cme}} t+r_{0 \mathrm{cme}}+d_{\mathrm{so}}, & t<\tau_{c 2}  \tag{15}\\ \frac{3}{2}\left(v_{0 \mathrm{cme}}-v_{1 \mathrm{AU}}\right) \tau_{c 2}\left[\left(\frac{t}{\tau_{c 2}}\right)^{2 / 3}-1\right] & \\ \quad+v_{1 \mathrm{AU}} \tau_{c 2}\left(\frac{t}{\tau_{c 2}}-1\right)+v_{0 \mathrm{cme}} \tau_{c 2}+r_{0 \mathrm{cme}}+d_{\mathrm{so}}, & t \geq \tau_{c 2}\end{cases}
$$

Equations (1), (5), (6) and (15) calculate speed and position of the CME and its shock.

## 3. Study Cases

We analysed seven fast ( $v_{0 \mathrm{cme}}>1000 \mathrm{~km} \mathrm{~s}^{-1}$ ) halo CME events during 1996-2009. The CMEs were associated with solar flares, and their IP shocks were related to type II kilometric radio bursts emissions. The events were reported in both LASCO (Gopalswamy et al., 2009) and Richardson and Cane (2010) lists. Table 1 shows the events and the initial conditions (and the solar wind at 1 AU ) that we used in our calculations.

We applied the model described in the previous section to calculate the CME leading edge and shock trajectories. We require as initial conditions: the CME position ( $r_{0}$ ) and speed ( $v_{0 \mathrm{cme}}$ ), and the flare rise time ( $\Delta t_{f}$ ). The solar wind conditions at 1 AU were defined by using average values of in-situ measurements by the Wind spacecraft around six hours upstream of the shock signatures. The polytropic index was $\gamma=1.5$. On the other hand, the initial density ratio $(c)$ of the CME /solar wind at the inner boundary was a free parameter, the value of which was selected to equal the CME arrival time (Corona-Romero and GonzalezEsparza, 2011).

We simulated the type II frequency drifts associated with the propagation of the IP shocks using the CME/shock trajectory for our four best events (see Table 1). These radio emissions are electromagnetic waves emitted by the solar wind electrons perturbed by an external agent. The radiation frequency or plasma frequency $\left(f_{\mathrm{p}}\right)$ is given by

$$
\begin{equation*}
f_{\mathrm{p}}(t)=\sqrt{\frac{e^{2} n_{1 \mathrm{AU}}}{4 \pi^{2} \epsilon_{0} m_{\mathrm{e}}}}\left[\frac{1 \mathrm{AU}}{r_{\mathrm{sh}}(t)}\right] \tag{16}
\end{equation*}
$$

Table 1 List of events and initial conditions. From left to right: number; date; CME time; CME-solar wind initial density ratio (Equation (4)); CME position; CME plane of the sky speed; solar flare rise time (CME injection time); in-situ solar wind values for bulk speed, density, temperature of protons, and magnetic field magnitude, respectively.

| Event |  |  | Initial conditions |  |  |  | ${ }^{\dagger}$ Solar wind (at $\sim 1 \mathrm{AU}$ ) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \# | *Date | *Hour [UT] | c | $\begin{aligned} & { }^{\star} r_{0} \\ & {\left[R_{\odot}\right]} \end{aligned}$ | ${ }^{\star} v_{0 \mathrm{cme}}$ $\left[\mathrm{km} \mathrm{s}^{-1}\right]$ | $\begin{aligned} & \dagger \Delta t_{f} \\ & {[\mathrm{~h}]} \end{aligned}$ | $\begin{aligned} & v_{1 \mathrm{AU}} \\ & {\left[\mathrm{~km} \mathrm{~s}^{-1}\right]} \end{aligned}$ | $\begin{aligned} & n_{1 \mathrm{AU}} \\ & {\left[\mathrm{~cm}^{-3}\right]} \end{aligned}$ | $\begin{aligned} & T_{1 \mathrm{AU}} \\ & {[\mathrm{kK}]} \end{aligned}$ | $\begin{aligned} & B_{1 \mathrm{AU}} \\ & {[\mathrm{nT}]} \end{aligned}$ |
| 1 | 2000-06-06 | 15:54 | 23.0 | 3.98 | 1119.3 | 0.40 | 510.0 | 4.3 | 180 | 5.9 |
| 2 | 2000-07-14 | 10:54 | 11.0 | 5.21 | 1674.0 | 0.45 | 600.0 | 2.5 | 120 | 4.5 |
| 3 | 2001-04-26 | 12:30 | 4.0 | 4.83 | 1006.0 | 1.50 | 440.0 | 2.1 | 60 | 6.0 |
| 4 | 2001-11-04 | 16:35 | 10.0 | 4.41 | 1810.0 | 0.23 | 330.0 | 3.3 | 15 | 7.0 |
| 5 | 2001-11-22 | 23:30 | 2.9 | 4.77 | 1437.0 | 1.40 | 430.0 | 5.5 | 160 | 7.0 |
| 6 | 2003-10-29 | 20:54 | 8.3 | 2.92 | 2029.1 | 0.54 | 450.0 | 2.0 | 150 | 9.0 |
| 7 | 2005-05-13 | 17:12 | 11.0 | 4.57 | 1689.0 | 0.40 | 410.0 | 3.5 | 110 | 5.5 |

* LASCO CME catalog (http://cdaw.gsfc.nasa.gov/CME_list/).
$\dagger$ GOES registers (http://www.swpc.noaa.gov/Data/goes.html).
$\ddagger$ Detected in-situ by Wind (http://omniweb.gsfc.nasa.gov/).
where $e$ is the fundamental charge, and $m_{\mathrm{e}}$ the electron mass. Given a value for $n_{1 \mathrm{AU}}$, Equation (16) relates a simulated radio emission $\left(f_{\mathrm{p}}\right)$ to the shock propagation $\left(r_{\mathrm{sh}}\right)$. We used this equation and its first harmonic ( $2 f_{\mathrm{p}}$ ) to compare it with the type II radio spectra detected by the WAVES experiment (Bougeret et al., 1995) on-board the Wind spacecraft.

Table 2 shows the comparison, for the seven CMEs, between the model results and the in-situ data at 1 AU . Columns 2 and 3 show the arrival speed of CMEs and shocks, and columns 4 and 5 show their TTs. Columns 6 and 7 show the critical times and distances ( $\tau_{c 1}$, $\tau_{c 2}, d_{c 1}, d_{c 2}$ ) indicating the driving and decaying phases in the heliocentric evolution of the CMEs and their shocks (Equations (2) and (7)).

### 3.1. Case Study 1: Event on 6 June 2000

This event was detected by LASCO/C2 on 6 June 2000 at 15:54 UT; it was associated with an X2.3 solar flare. The CME reached 1 AU on 8 June at 12.00 UT. Figure 2a shows the CME and shock speed evolution as obtained from the model. At the beginning (driving phase), the CME (thin curve) and its shock (thick curve) propagate with a constant speed equal to the LASCO linear fit (open diamond). This constant speed lasts until $\tau_{c 1}$ (vertical dashed line), after which the CME decelerates. The shock speed is constant until $\tau_{c 2}$ (vertical dotted line); for longer times, the shock decelerates (decaying phase). Note that the calculated CME and shock arrival speeds (solid circumferences) are very close to the in-situ measurements. Figure 2 b shows the CME and shock trajectories as given by the model. At the beginning, the CME matches the LASCO data (diamonds). As commented on before, due to the arbitrary selection of $c$, the calculated CME transit time (open circle) matches the in-situ measurement (cross); the calculated shock arrival (open circle) occurs two hours earlier than the in-situ register (plus sign). Figure 2c shows the Wind/WAVES (Bougeret et al., 1995) dynamic spectrum during the period of time where the event is propagating. We notice a type II radio burst, drifting from about 14 MHz down to 40 kHz . At the beginning of the emission, a fundamental-harmonic ( FH ) pair is observed; then a single tone, sometimes very intense, is drifting to lower frequencies. Overplotted on the dynamic spectrum

Table 2 Comparison of in-situ measurements and model results. (1) CME front and shock speeds at 1 AU ; (2) CME front and shock transit times; (3) CME/shock critical times ( $\tau$ ) and critical distances (d).

| Event | (1) Arrival speeds |  | (2) Transit times (TT) |  | (3) Driving and decaying |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & v_{\mathrm{cme}} \\ & {\left[\mathrm{~km} \mathrm{~s}^{-1}\right]} \end{aligned}$ | $\begin{aligned} & v_{\mathrm{sh}} \\ & {\left[\mathrm{~km} \mathrm{~s}^{-1}\right]} \end{aligned}$ | CME <br> [h] | shock <br> [h] | $\begin{aligned} & \overline{\tau_{c 1}-d_{c 1}} \\ & {[\mathrm{~h}]-\left[\mathrm{R}_{\odot}\right]} \end{aligned}$ | $\begin{aligned} & \tau_{c 2}-d_{c 2} \\ & {[\mathrm{~h}]-\left[\mathrm{R}_{\odot}\right]} \end{aligned}$ |
| 1 | 770^/795 | $871^{\dagger} / 865$ | 44.1*/44.3 | $41.3^{\star} / 39.4$ | 3.9-26.7 | 7.9-49.8 |
| 2 | 1000^/1035 | $1120^{\dagger} / 1153$ | 32.1*/32.1 | 27.7*/29.1 | 2.7-28.8 | 4.1-40.6 |
| 3 | $700^{\star} / 688$ | $812^{\dagger} / 832$ | 49.5*/49.7 | 40.5 */39.6 | 6.8-40.4 | 13.6-75.8 |
| 4 | 730 * $/ 660$ | \#/881 | 43.6 */45.5 | $33.4 * / 35.6$ | 1.1-15.3 | 1.9-22.7 |
| 5 | $700^{\star} / 823$ | 1008 ${ }^{\dagger} / 994$ | $37.5^{\star} / 38.54$ | $30.4^{\star} / 33.3$ | 4.7-39.6 | 6.2-50.1 |
| 6 | 1040 * $/ 1074$ | \#/1264 | 28.9*/29.1 | $21.22^{\star} / 25.4$ | 2.54-29.6 | 3.69-41.7 |
| 7 | 900^/848 | $942^{\dagger} / 982$ | $36.8^{\star} / 37.0$ | $33.4{ }^{\star} / 33.0$ | 2.2-23.4 | 3.2-32.6 |

* Wind measurements and Richardson and Cane (2010) catalogue.
${ }^{\dagger}$ Calculated using the velocity coplanarity (see the Appendix, Equation (22)) and Wind in-situ data.
\# Data gap.
Analytical results: $d_{c 1}=v_{0 \mathrm{cme}} \tau_{c 1}+r_{0}$, and $d_{c 2}=v_{0 \mathrm{cme}} \tau_{c 2}+r_{0}$.
are the calculated fundamental plasma frequency (solid line) and its first harmonic (dashed line). We see then that the first harmonic is qualitatively consistent with the lower part of the emission band.


### 3.2. Case Study 2: Event on 14 July 2000

The Bastille day event appeared in LASCO/C2 on 14 July 2000 at 10:54 UT, and it was associated with a X5.7 flare. The CME arrived at 1 AU on 15 July at 19:00 UT. This event is particularly difficult to study due to the multiple CME arrivals from 10 July to 15 July; in fact, another CME passed by the Earth neighbourhood (15:32 UT) just a few hours before the event arrival (Richardson and Cane, 2010). These circumstances complicated the acquisition of solar wind data and affected the CME/shock evolution by generating interactions between the CME and the previous disturbances. Nevertheless, the general results are similar to those obtained in the previous case showing quantitative and qualitative agreement with data. Figure 3a shows the evolution of CME and shock speeds as inferred from the model. We can appreciate the two phases (driving-decaying) in both speed profiles. The critical times ( $t_{c 1}$ and $t_{c 2}$ ) are shorter than in the previous case (see Table 2). The calculated CME arrival speed matches with the in-situ data; whereas the analytical shock arrival speed is faster than its in-situ counterpart. Figure 3b presents the CME leading edge and shock positions. Close to the Sun, the CME leading edge matches the LASCO data (diamonds). The estimated CME transit time (open circle) matches the in-situ measurement (cross); the calculated shock arrival (open circle) occurs about one hour earlier than the in-situ measurement (plus sign). Figure 3c shows the dynamic spectrum associated with the event. We notice that the type II radio burst is contaminated with a type III storm as a result of the solar activity, before and after the event. Similar to the previous case study, the calculated first harmonic (dashed line) is qualitatively consistent with the lower part of the emission band.

Figure 2 Case study 1: comparison of model results and observations. (a) Evolution of CME (thin line) and shock speeds (thick line) as deduced from the analytical model. Vertical dashed and dash-dotted gray lines indicate $\tau_{c 1}$ and $\tau_{c 2}$, respectively (see Table 2). (b) CME (thin line) and shock (thick line) trajectories as deduced from the model. The symbols indicate the in-situ measurements from the Wind spacecraft, and the horizontal dotted line indicates 1 AU . (c) Type II dynamic radio spectra detected by Wind/WAVES, and computed fundamental plasma frequency (solid line) and first harmonic (dashed line) as deduced from the model (see text).


### 3.3. Case Study 3: Event on 26 April 2001

This event was detected by LASCO/C2 on 26 April 2001 at 12:30 UT, and it was associated with an M7.8 X-ray solar flare. The CME arrived at 1 AU on 28 April at 14:00 UT. Figure 4a shows the two propagation stages, we also appreciate coincidences between the radio data and the calculated shock propagation. The calculated arrival speeds for the CME leading edge and its shock are very close to their in-situ counterparts. Figure 4b shows that the CME leading edge position matches with most of the LASCO data, and the calculated shock TT is quite similar to the in-situ measurements (less than one hour of difference). Figure 4 c shows the dynamic spectrum during the period of time when the event is taking place. We notice that the type II radio burst is extremely chaotic, drifting from about $\sim 5 \mathrm{MHz}$ down to $\sim 20 \mathrm{kHz}$. Moreover, there is a $\sim$ nine hour gap in the dynamic spectrum on 26 April. Overplotted on the dynamic spectrum are the calculated fundamental plasma frequency (solid line) and its first harmonic (dashed line). We see that the first harmonic is qualitatively consistent with the lower part of the type II emission band.

Figure 3 Case study 2: Bastille day event 2000, comparison of model results and observations. Same format as in Figure 2.


### 3.4. Case Study 4: Event on 13 May 2005

This event was detected in LASCO/C3 on 13 May 2005 at 17:12 UT and was associated with a M8.0 flare; the CME arrived at 1 AU in 15 May at 06:00 UT. Figure 5 shows the two phases in the CME and shock speeds. We also observe that the CME and shock evolutions are consistent with coronagraph and in-situ data. The arrival speeds and TTs of the CME and its shock are quantitative similar to the in-situ values. Figure 5c shows that the calculated first harmonic closely follows the lower part of the type II emission associated with the shock, from the beginning of the event up to 02:00 UT on 15 May.

This event was analysed in detail by Bisi et al. (2010), who report a CME/shock deceleration as the event evolved from near the Sun to 1 AU . Figure 5 shows qualitative agreement with this interpretation. However, a direct comparison between our analytic results and the speed data points (Figure 35 in Bisi et al., 2010) is difficult. The speed analysis by Bisi et al. presents a significant dispersion, which, as the authors point out, might be due to the fact that the speed data points were associated with different regions at different times of the CME/shock event.

Figure 4 Case study 3:
comparison of model results and observations. Same format as in Figure 2.


## 4. Discussion

We present an analytical model for approximate fast CME/shock propagation based on the dynamics of an ejecta driver and a driven shock wave. According to our model there are two main stages:
i) The driving phase ( $0<t<\tau_{c 1}$ ), when the CME drives the shock wave and the separation between the CME leading edge and its shock is about constant.
ii) The decaying phase ( $t>\tau_{c 2}$ ), when the CME tends to equal the solar wind speed, the shock wave evolves into a blast wave, and the separation between the CME leading edge and the shock front increases.

As initial conditions, our model requires: CME initial speed ( $v_{0 \mathrm{cme}}$ ), position ( $r_{0 \mathrm{cme}}$ ), density ( $n_{0 \mathrm{cme}}$ ), injection time ( $\Delta t_{f}$ ) and the ambient solar wind conditions. In order to apply our model to study fast CMEs propagating to the Earth, we used $v_{0 \mathrm{cme}}$ and $r_{0 \mathrm{cme}}$ from coronagraph images (LASCO observations), $\Delta t_{f}$ from the flare rise phase (soft X-ray fluxes), and the solar wind conditions from in-situ measurements at 1 AU assuming a radial

Figure 5 Case study 4:
comparison of model results and observations. Same format as in Figure 2.

expansion. The initial CME density (Equation (4)) is our only free parameter, which was chosen to equal the in-situ CME transit time.

The initial CME density ( $n_{0 \mathrm{cme}}$ ), expressed by $c$ in Equation (4), is a free parameter and it runs over a wide range of values (see Table 1). Large $c$ values are associated with CMEs with large inertia, decreasing the SW effects on the CME dynamics. Thus, the larger the $c$ value, the shorter the CME transit time (TT) and faster the arrival CME speed at 1 AU . Large values of $c$ also have an impact on the shock propagation, causing shorter TTs and faster arrival shock speeds. The driving phase duration (Equation (2)) increases with $c$.

The value of $n_{0 \mathrm{cme}}$ is difficult to approximate from the current observations, because it implies that one should know the coronal structure, the local density of the SW, the CME geometry and its total mass. The CME total mass has been estimated by employing Thomson scattering on coronagraph observations (Vourlidas et al., 2000; Colaninno and Vourlidas, 2009) and also using extreme ultra-violet dimming (Aschwanden et al., 2009). The total mass is related to Equations (6) and (18) and gives us a parameter to evaluate our initial conditions. In general, the $c$ values that we used in Table 1 agree with the orders of magnitude of the CME total masses reported in studies commented on before.

The two-stages propagation for a fast CME/shock is consistent with empirical and numerical studies (e.g. Manoharan, 2006; Pohjolainen et al., 2007 and González-Esparza et al., 2003c). The analytic description for the shock evolution is dynamically and mathematically similar to the semi-empirical relation proposed by Pinter and Dryer (1990) for shocks associated with solar flares.

We discussed four fast halo CMEs (Earth-directed) simulating the type II radio drift emission associated with the IP shock. In all the cases, the calculated CME initial trajectories were consistent with LASCO data. The calculated arrival speeds and transit times of CMEs and shocks were quantitatively consistent with their in-situ measurements at 1 AU . According to our case study results (Table 2), the differences between the calculated and measured CME arrival speeds were less than $6 \%$; the differences for both shock arrival speeds and transit times were less than $5 \%$.

In our analytical model the CME injection time ( $\Delta t_{f}$ ) represents the period when the CME acquires its initial kinetic energy and linear momentum (Cantó, Raga, and D’Alessio, 2000). We approximated this parameter as the flare rising phase, because that interval is associated with the time when the energetic CME reaches an almost constant speed (kinetic energy) in the coronagraph field of view (Zhang and Dere, 2006). Although it is widely accepted that CMEs are not necessarily related to solar flares (Gosling, 1993), some studies relate solar flares to the initiation of energetic CMEs. For example, Zhang et al. (2004) and Zhang and Dere (2006) found some relationships between soft X-ray fluxes and CME initial accelerations. On the other hand, Temmer et al. (2008) found a close synchronisation between the CME acceleration profiles and flare hard X-ray flux onsets. Furthermore, Chen and Kunkel (2010) found that the observed duration of soft X-ray emission is comparable to the poloidal flux injection, and such injection is related to the CME initial acceleration (Chen, 2001). Although in these studies a possible relation between the CME and flare initiation mechanisms appears, the authors also conclude that such relation is not trivial and requires further research. Since we are not aware of any observation that could directly provide us with the CME injection time, we choose the flare rising time as an equivalent; since, in our case, these energetic CMEs were associated with solar flares.

Our results suggest relations between the CME initial properties and the associated flare. Figure 6 shows the initial conditions for the seven fast CME events listed in Tables 1 and 2. Figure 6a presents the initial CME density with respect to the ambient wind (c) versus the flare rising time $\left(\Delta t_{f}\right)$. The data points show the tendency that the flare rising time is inversely proportional to the CME density jump. In Figure 6b, the jump in CME-solar wind kinetic energy ( $c v_{0 \mathrm{cme}}^{2} / v_{1 \mathrm{AU}}^{2}$ ) decreases as $\Delta t_{f}$ increases:

$$
\begin{align*}
c & =5.21\left[\frac{\Delta t_{f}}{1 \mathrm{~h}}\right]^{-0.82}  \tag{17}\\
c \frac{v_{0 \mathrm{cme}}^{2}}{v_{1 \mathrm{AU}}^{2}} & =44.18\left[\frac{\Delta t_{f}}{1 \mathrm{~h}}\right]^{-1.31} . \tag{18}
\end{align*}
$$

Although these tendencies were consistent for all the analysed events, we need to study a larger number of cases in order to corroborate the results. The trends in Equations (17) and (18) are in agreement with results of Zhang and Dere (2006). Zhang and Dere (2006), after having studied the initial acceleration of CMEs from coronagraph images, found that the stronger the "main" CME acceleration near the Sun $\left(a_{0 \mathrm{cme}}\right)$, the shorter the duration $\left(\Delta t_{a}\right)$ of such acceleration. This relationship is expressed by $a_{0 \mathrm{cme}} / 1 \mathrm{~m} \mathrm{~s}^{-2}=135.46\left(\Delta t_{a} / 1 \mathrm{~h}\right)^{-1.09}$, where $\Delta t_{a} \approx \Delta t_{f}$. Consequently, impulsive events (short $\Delta t_{f}$ ) tend to have more inertia and faster accelerations (i.e. larger relative kinetic energies). On the other hand, for gradual

Figure 6 Relationships of the CME initial conditions and the solar flare rise time ( $\Delta t_{f}$ ) of the events in Table 1. (a) Ratio between the CME and solar wind initial densities (Equation (4)) versus $\Delta t_{f}$. (b) Ratio between the CME and solar wind initial kinetic energies versus $t_{f}$. Open squares represent the four case studies. Solid lines are the best fit to the data.

events (long $\Delta t_{f}$ ) we expect the opposite tendency. This suggests that $\Delta t_{f}$ might be related to the physical mechanisms by which a fast CME acquires its initial kinetic properties, in agreement with Zhang et al. (2004).

In this study we select the value of $c$ to equal the calculated and reported transit times of CMEs. However, as we commented on before, $c$ is difficult to measure directly from observations and we need an indirect method to estimate the initial kinetic properties of CMEs. In this sense, Equations (17) and (18) may be useful to approximate or to delimit the initial values for CME density and kinetic energy. Thus, it is important to develop further studies in order to corroborate or discard the mentioned relations.

In the seven events, the decaying stage began long before the shock reached 1 AU $\left(\sim 50 R_{\odot} \approx 0.25 \mathrm{AU}\right.$; see Table 2). Based on our model, this means that IP shocks associated with fast CMEs evolve like blast waves during most of their transit time to 1 AU . This result agrees with other studies. For example, Burlaga et al. (1981) pointed out, discussing linear momentum fluxes, that some CMEs may not drive shocks any more at distances around 2 AU . On the other hand, Feng et al. (2010) analysed the geometric properties of CME and shocks at 1 AU and concluded that at least $34 \%$ of all shocks were not driven by their associated CMEs. Maloney and Gallagher (2011) measured standoff distances of fast CME/shocks within heliocentric ranges between $2-120 R_{\odot}$ and, in general, found them to be larger than expected. These results agree with an early onset of the decaying stage in CME/shock propagation.

We reproduce the radio frequency drift associated with the propagation of the IP shock. There are other studies comparing the evolution of CME/shock events and type II radio burst spectra detected by Wind/WAVES. Reiner, Kaiser, and Bougeret (2007) used an empirical model to calculate the arrival times of CME/shocks, assuming an initial strong deceleration followed by a constant speed propagation of the CME/shock (this constant speed is much faster than the ambient solar wind). Lara and Borgazzi (2009) assumed that the radio emission was associated with the propagation of the CME mass centre, the evolution of which
was affected by viscosity and drag forces of the solar wind. What is different in our model is that we solve specifically both the CME and shock propagation. Then we calculate the type II frequency drift emission from the shock propagation solution. The CME and its shock suffer different heliocentric evolution, and propagate at different speeds. This becomes more significant when the magnetosonic Mach number is small, resulting in larger standoff distances and greater differences between the CME and the shock speeds and positions.

In all the case studies, the first harmonic of the simulated radio-burst emission agreed qualitatively with the lower part of the radio spectra detected by Wind/WAVES. This is somewhat similar to the study by Reiner, Kaiser, and Bougeret (2007), who used the first harmonic to adjust the CME/shock trajectories. However, in our case we match the lower part of the radio spectra. This former result is consistent with the study by Knock and Cairns (2005), who showed that the plasma frequency is a lower limit for the type II radio-burst emission, and the width of the spectra is related to the shock expansion and solar wind fluctuations. We do not take into account these two aspects in our model.

## 5. Conclusions

We analysed four fast CME halo events as case studies using an analytical model and compared the results with different data. The model implies two propagation stages for the CME and its shock. The dynamic processes between the CME and its shock wave are important to understand their heliocentric evolution. To perform the calculations we used different observations as initial conditions. In general, we found good agreement comparing the results and the CME/shock in-situ data at 1 AU . The first harmonic of the simulated radio-burst emission associated with the shock propagation was consistent with the lower part of the type II radio spectra detected by Wind/WAVES. The results of the analytical model imply that the shocks were not driven anymore by their CMEs when they reached 1 AU .

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## Appendix

## A. 1 Polytropic MHD Jump Relations

Polytropic MHD jump relations are the specific jump relations used in this work; for a more general solution see Petrinec and Russell (1997), Equations (14), (15) and (16). The downstream variables (subindex 2 ) are related with their upstream counterparts (subindex 1) according to

$$
\begin{align*}
& \frac{1}{n_{*}}=\frac{\rho_{1}}{\rho_{2}}=-\frac{k_{1}}{k_{9}}-\frac{2^{1 / 3} k_{3}}{\mu_{2}^{2} k_{8} k_{9}}+\frac{k_{8}}{2^{1 / 3} \mu_{2}^{2} k_{9}},  \tag{19}\\
& B_{*}=\frac{\left|B_{2}\right|}{\left|B_{1}\right|}=\sqrt{c_{\theta 2}\left(1-\alpha^{2}\right)+\alpha^{2}}  \tag{20}\\
& p_{*}=\frac{p_{2}}{p_{1}}=1+\left(1-\frac{1}{n_{*}}\right) \frac{2 \mu_{2}}{\beta}+\frac{1-c_{\theta 2}}{\beta}\left(1-\alpha^{2}\right) . \tag{21}
\end{align*}
$$

In Equations (19), (20) and (21) we have used

$$
\begin{aligned}
c_{\theta 2} & =\cos ^{2}\left(\theta_{B n}\right), \\
\mu_{2} & =M_{A 1}^{2}, \\
\beta & =2 \mu_{0} \frac{p_{1}}{B_{1}^{2}}, \\
\alpha & =\frac{c_{\theta 2}-\mu_{2}}{c_{\theta 2}-\mu_{2} / n_{*}}, \\
k_{1} & =-\gamma(1+\beta)+\mu_{2}(1-\gamma)-c_{\theta 2}(2+\gamma), \\
k_{2} & =\mu_{2}(-2+\gamma)+c_{\theta 2}\left(1+\gamma\left[1+2 \beta+\mu_{2}\right]\right), \\
k_{3} & =\mu_{2}^{4}\left(-k_{1}^{2}+3[1+\gamma] k_{2}\right), \\
k_{4} & =\mu_{2}(1-\gamma)-\beta \gamma c_{\theta 2}, \\
k_{5} & =9(1+\gamma) \mu_{2}^{6} k_{1} k_{2}, \\
k_{6} & =-2 \mu_{2}^{6} k_{1}^{3}-27(1+\gamma)^{2} \mu_{2}^{6} c_{\theta 2} k_{4}+k_{5}, \\
k_{7} & =\left(4 k_{3}^{3}+k_{6}^{2}\right)^{1 / 2}, \\
k_{8} & =\left(k_{6}+k_{7}\right)^{1 / 3}, \\
k_{9} & =3(1+\gamma) \mu_{2} .
\end{aligned}
$$

To obtain Equations (19), (20) and (21) we assume that the normal to the shock is radial at the shock front.

## A. 2 Velocity Coplanarity

The velocity coplanarity is commonly used to approximate the shock velocity by applying the mass conservation at the shock reference frame. If a shock wave propagates with a velocity $\mathbf{v}_{\text {sh }}$ through an ambient solar wind with density $\rho_{1}$ and velocity $\mathbf{v}_{1}$, the shock wave velocity shall fulfil $\rho_{1}\left(\mathbf{v}_{\text {sh }}-\mathbf{v}_{1}\right)=\rho_{2}\left(\mathbf{v}_{\text {sh }}-\mathbf{v}_{2}\right)$, where the subindex 2 indicates the downstream values. Thus, solving for $\mathbf{v}_{\text {sh }}$ :

$$
\begin{equation*}
\mathbf{v}_{\mathrm{sh}}=\frac{\rho_{2} \mathbf{v}_{2}-\rho_{1} \mathbf{v}_{1}}{\rho_{2}-\rho_{1}} \tag{22}
\end{equation*}
$$

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