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Precise Determination of the Orientation of the Solar Image

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Abstract Accurate heliographic coordinates of objects on the Sun have to be known in several fields of solar physics. One of the factors that affect the accuracy of the measurements of the heliographic coordinates is the accuracy of the orientation of a solar image. In this paper the well-known drift method for determining the orientation of the solar image is applied to data taken with a solar telescope equipped with a CCD camera. The factors that influence the accuracy of the method are systematically discussed, and the necessary corrections are determined. These factors are as follows: the trajectory of the center of the solar disk on the CCD with the telescope drive turned off, the astronomical refraction, the change of the declination of the Sun, and the optical distortion of the telescope. The method can be used on any solar telescope that is equipped with a CCD camera and is capable of taking solar full-disk images. As an example to illustrate the method and its application, the orientation of solar images taken with the Gyula heliograph is determined. As a byproduct, a new method to determine the optical distortion of a solar telescope is proposed.

Keywords Heliographic coordinates · Sun · Sunspots

1. Introduction

Accurate heliographic coordinates of objects on the Sun have to be known in order to study their movement relative to the solar surface. Such movements are used, for example, to study solar differential rotation (Gilman and Howard, 1984), meridional motion (Howard and Gilman, 1986), and proper motion of sunspots (Gesztelyi and Kálmán, 1986). In certain cases (see Kennedy, 1997), helioseismology measurements require an accuracy better than $0.02°$ in the solar image orientation.

Information on the heliographic coordinates is also useful when we want to superimpose solar images of different kinds (white light, Hα, calcium, and magnetic, to name only a few) taken with different telescopes to study the relationship between them. The simplest method

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to place one image over another is by using the heliographic coordinates of representative image points. For example, if we want to superimpose a magnetogram and a white light image in order to determine the magnetic field of a small sunspot, we must know the heliographic positions of both images with an accuracy higher than 0.1 heliographic degrees, since the diameter of a small sunspot is about 0.1°.

When compiling the DPD (Debrecen Photoheliographic Data) sunspot catalogue as a continuation of the Greenwich Photoheliographic Results, we use observations from several observatories to fill the gaps in our observations. We often notice that the heliographic sunspot coordinates disagree among different observatories. The difference can be as high as one heliographic degree. The deviations generally depend on the heliographic position of the sunspot and can be explained by the mis-orientation of the solar images.

The accuracy of the heliographic coordinates is closely tied to the accuracy to which we know the orientation of the solar image on the detector. Therefore, we determine how the orientation error of the solar image propagates into the heliographic coordinates. The propagation of the orientation error $\Delta\tau$ for heliographic longitude L and heliographic latitude B can be written as follows:

$$\Delta L = (\sin B_{\mathrm{o}} - \cos B_{\mathrm{o}} \tan B \cos L_{\mathrm{cm}})\Delta\tau, \tag{1}$$

$$\Delta B = \cos B_{\mathrm{o}} \sin L_{\mathrm{cm}} \Delta\tau, \tag{2}$$

where B_{o} is the heliographic latitude of the center of the solar disk and L_{cm} is the central meridian distance corresponding to L. These formulas show two important features of the error propagation:

- The maximum error in the heliographic coordinates caused by the alignment error is about equal to the alignment error itself.
- The error depends on the heliographic position, particularly on the central meridian distance. Therefore, it may lead to an incorrect proper motion of the solar features during their lifetime. For example, let us consider a sunspot that does not move, *i.e.*, its heliographic coordinates do not change. But if we have systematic orientation error of the solar images taken during its lifetime, then its measured heliographic coordinates will change according to Equations (1) and (2) as its central meridian distance changes.

When the media to record solar images was film, one of the methods that were used to determine the orientation of the solar image on the film was the so-called drift method. In this method, a thread located very near the image plane of the telescope is adjusted (rotated) by the observer so that the trajectory of a sunspot when the telescope drive is turned off should be on this line. The direction of the thread, so adjusted, is an approximation of the geocentric east – west direction of the solar image. It must be stressed that this is only an approximation, because this method does not make the corrections needed for other effects (refraction, the change of the Sun's declination arising from the tilt of the equator with respect to the ecliptic, and the inherent deviation of the trajectory from a straight line). Moreover, this method is sensitive to human errors.

Nowadays film is replaced by CCD cameras and the drift method is suitable for such a digital recording device. Taking a series of solar images with the CCD camera by turning off the telescope drive, the trajectory of the center of the solar disk, after proper corrections, represents the geographic east – west direction of the solar image on the detector. Figure 1 shows such a series, where the solar images in this series were taken three seconds apart.

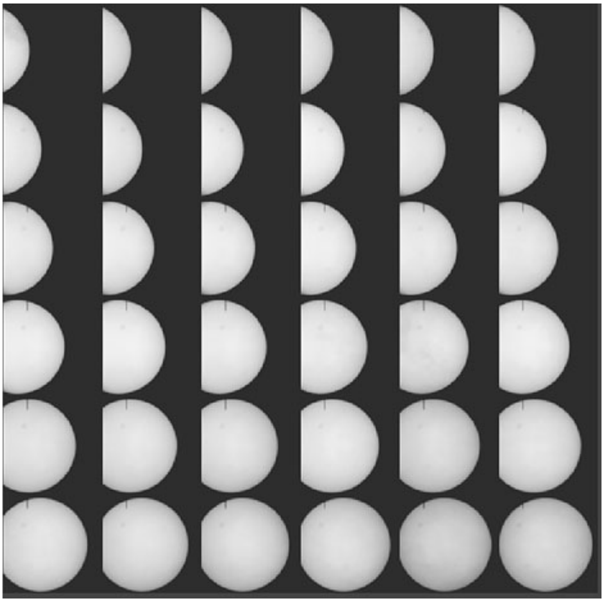

Figure 1 Thumbnail version of a series of solar images taken as the solar disk transits across the CCD detector.

2. Corrections

In this section, we describe how to determine the centers of an image series to find the geocentric east – west direction of the image plane.

2.1. Ideal Trajectory

When the telescope drive is turned off, the center of the solar disk leaves a trajectory across the primary image plane of a telescope. In an ideal case, when we neglect two types of the change of the Sun's declination, namely the one caused by the astronomical refraction and the other by the tilt of the equator with respect to the ecliptic, this trajectory is determined by the intersection of the image plane and a cone. The above statement is valid as long as the declination of the center of the solar disk is not zero. If it is zero, then the trajectory of the solar disk center is a straight line determined by the intersection of the image plane with the plane of the equator. In the case of the Sun, the cross-section of this cone can be approximated, with sufficient accuracy, by the following equation:

$$y = \frac{r_{\mathrm{s}} \tan(\delta_{\mathrm{s}} - \delta_{\mathrm{t}})}{\tan \alpha_{\mathrm{s}}} + \frac{x^2 \tan \delta_{\mathrm{s}} \tan \alpha_{\mathrm{s}}}{2 r_{\mathrm{s}}}, \tag{3}$$

where δ_{s} is the declination of the solar disk center, δ_{t} is the declination of the telescope, α_{s} is the angular semi-diameter of the solar disk, and r_{s} is the radius of the solar image. Equation (3) holds in such a coordinate system where the y-axis is the geocentric north direction of the telescope's image plane.

2.2. Effects Arising from the Change of the Sun's Declination

As the solar image passes across the CCD detector, while the telescope drive is turned off, the position of the image center in the north – south direction (*i.e.*, the y-coordinate of the center) changes too, due to the change in its declination caused by the combined effects of the tilt of the equator with respect to the ecliptic and the revolution of the Earth around the Sun.

This change, denoted by Δy_δ, relative to the first image can be described as

$$\Delta y_\delta = S(\delta_{si} - \delta_{s1}), \tag{4}$$

where δ_{s1} and δ_{si} are the Sun's declination when the first and the i-th image in the series were taken, respectively. The parameter S refers to the image scale.

2.3. Influence of the Terrestrial Atmosphere on the Full-Disk Image

During the passage of the solar disk across the CCD detector, its center is diverted by atmospheric refraction from its course along the curve given by Equation (3) toward the nadir (before the local noon) or the zenith (after the local noon). In addition, the refraction distorts the solar disk, particularly at low altitude.

2.3.1. Refraction Changes the Observed Declination of the Sun

As the refraction moves the center of the solar disk toward the zenith, its observed declination also changes. The change of the declination $\Delta\delta_r$, caused by the refraction while the solar image drifts through the telescope's image plane, can be described by the following equation (see *e.g.*, Smart, 1977):

$$\Delta\delta_r = R \cos q, \tag{5}$$

where R is the refraction and q is the parallactic angle, *i.e.*, the angle between the great circles connecting the center of the solar disk with the zenith and the north pole.

Using Equation (4), the shift Δy_r of the solar disk center along the north – south direction on the i-th image of the series can be given as

$$\Delta y_r = S(R_i \cos q_i - R_1 \cos q_1). \tag{6}$$

Note that the shift Δy_r caused by the refraction is given relative to the first image in the series.

2.3.2. Refraction Distorts the Solar Disk

Our objective here is to find the true center of the solar image. The solar limb points should lie on a circle. Thus, fitting a circle to them, the center of the solar disk could be determined. However, when a solar image is taken through the Earth's atmosphere, it becomes distorted as a consequence of differential refraction. Therefore, this simple approach cannot be used.

In this distorted image, we need to find the point that corresponds to the true center of the solar disk. The following procedure yields the true center with an accuracy of $0.5''$, even at a zenith distance as large as $87°$.

- The image is stretched along the vertical direction by the factor s given by

$$s = \frac{1}{1 - \frac{dR}{dZ}|_{Z=Z_c}} = 1 + \frac{dR}{dZ_o}\bigg|_{Z_o=Z_{oc}}, \tag{7}$$

  where R is the refraction, Z is the true zenith distance, Z_o is the observed zenith distance, Z_c is the true zenith distance of the solar disk center, and Z_{oc} is the observed zenith distance of the solar disk center.

- The center of the stretched image is determined by fitting a circle to the limb points.
- The center of the stretched image is shifted, opposite to the zenith, by the value Δr given by

$$\Delta r = \frac{\frac{\mathrm{d}^2 R}{\mathrm{d}Z^2}|_{Z=Z_\mathrm{c}}}{1 - \frac{\mathrm{d}R}{\mathrm{d}Z}|_{Z=Z_\mathrm{c}}} r_\mathrm{m}^2 = \frac{\mathrm{d}^2 R}{\mathrm{d}Z_\mathrm{o}^2}\bigg|_{Z_\mathrm{o}=Z_\mathrm{oc}} r_\mathrm{m}^2, \tag{8}$$

  where r_m is the measured radius of the stretched image.

2.4. Optical Distortion of the Solar Telescope

Generally, a solar telescope has magnifying optics in addition to its objective. The magnifier may have optical distortion. To be more precise, we use the term optical distortion here as one of the Seidel aberrations (spherical aberration, coma, astigmatism, curvature of field, and distortion). If r_o and r_1 denote the distances measured from the optical axis in the primary and the secondary image planes of the telescope, respectively, then, in the case of distortion, the relation between them is given by Born and Wolf (1965)

$$r_1 = M r_\mathrm{o} + E r_\mathrm{o}^3, \tag{9}$$

where M is the enlargement factor for paraxial rays (Gaussian magnification) and E a constant depending on the magnifier. Since E is small, we have from Equation (9)

$$r = \frac{r_1}{1 + C r_1^2}, \tag{10}$$

where $r = M r_\mathrm{o}$ and $C = E/M^3$. Equation (10) transforms the image plane of the magnifier of the telescope into an image plane without distortion and having the paraxial enlargement of the original magnifier as magnification. It can be used to correct for the optical distortion in the solar image, if the constant C is known.

If the center of a circle (the solar limb) is located on the optical axis, then the circle is not distorted because the distortion only depends on the distance from the optical axis, *i.e.*, only its radius will be changed by the distortion. But if the center of the circle is not on the optical axis, then the circle becomes distorted because of its asymmetrical location with respect to the optical axis and its distortion depends on the position of its center. Now, if we fit a circle to the limb points, the fitted radius will also depend on the position of its center.

Our procedure to correct for the distortion is as follows. First, a function is constructed by the following algorithm using the solar images obtained by the drift method:

- After correcting the i-th image in the series for refraction, the radius (r_i) and the center coordinates (u_i, v_i) are determined.
- The distortion effect on the circle with radius r and center (u_i, v_i) are simulated. Several points on this circle are calculated, but only those points on the CCD detector are considered. The circle points are displaced by the amount r_1 given by Equation (10). By fitting a circle to the displaced points, the distorted radius $r_{\mathrm{s}i}$ is obtained.
- The function L is defined by

$$L(r, C) = \frac{1}{n} \sum_{i=1}^{n} \left[r_{\mathrm{s}i}(r, C) - r_i \right]^2, \tag{11}$$

  where n is the number of images used in the calculation.

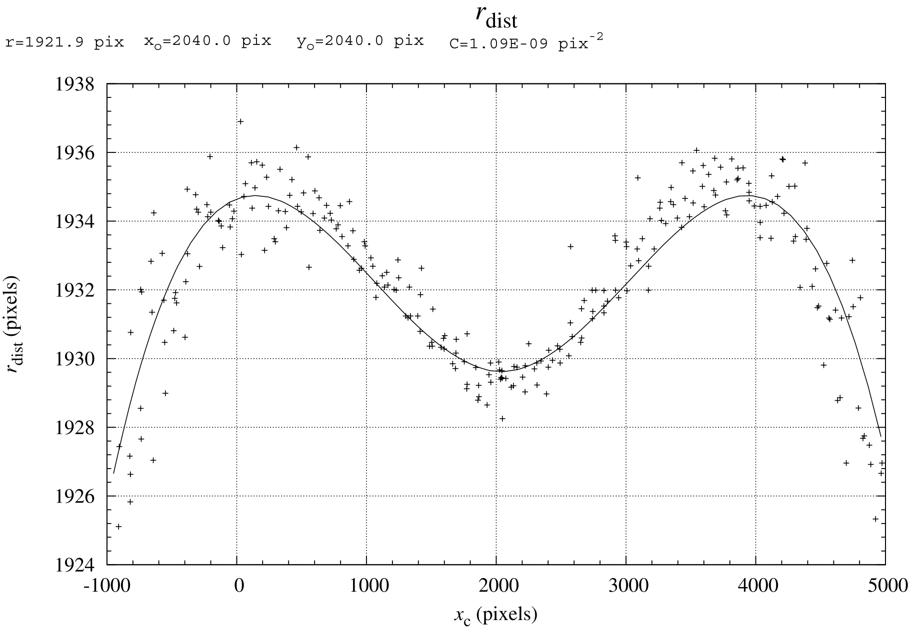

Figure 2 The simulated distorted (solid curve) and the measured (+) radii of the solar image as a function of the x-coordinate of the solar disk center for the Gyula heliograph.

Since L is the average square difference between the simulated and measured radii, given as a function of the undistorted radius r and the distortion parameter C, these parameters can be obtained by minimizing L with respect to r and C.

The simulated distorted and measured radii of the solar image for the Gyula heliograph are presented in Figure 2 as a function of the x-coordinate of the solar disk center. The simulated distorted radii are calculated with the parameters $r = 1921.9$ pixels and $C = 1.09 \times 10^{-9}$ pixel^{-2} that minimize the function $L(r, C)$. The coordinates (in pixels) of the optical center of the image are (2040.0, 2040.0).

3. An Iterative Approach for Determining the Orientation of the CCD Camera

If we correct the solar limb for refraction and optical distortion, and apply the corrections implied by Equations (3), (4), and (6) to the trajectory of the solar disk center, we obtain a straight line representing the geocentric east – west direction of the solar image. However, to perform these corrections we should know this direction in advance, because Equations (3), (4), and (6) hold only in a coordinate system whose x-axis is the geocentric east – west direction. Furthermore, for the refraction correction of the solar limb, we should know the vertical direction of the solar image, which can also be determined from the east – west direction. Therefore, an iterative approach must be applied:

i) A straight line is fitted to the trajectory of the disk center in the CCD coordinate system to obtain a preliminary east – west direction. Here no corrections are applied at all.

ii) The coordinate system is rotated by the inclination angle of the straight line, and the corrections are performed.

iii) The coordinate system is rotated back, and a straight line is fitted to the corrected trajectory of the disk center.
iv) Steps *ii*) and *iii*) are repeated until the change in the angle of the fitted straight line becomes small enough.

As the corrections are small quantities, two iterations generally results in an east – west direction with sufficient accuracy. This rotation angle ΔP_o between the CCD detector and the geocentric coordinate system (*i.e.*, the angle between the rows of the CCD matrix and the geocentric east – west direction) is used in the following sections.

4. Summary of the Correction Algorithm

So far we have described the means needed to determine how a CCD camera attached to a telescope can be aligned. Now the process to determine the orientation of the CCD can be summarized in the following steps:

- For every image in a drift series,
  - the solar limb pixels are found;
  - the limb pixels are corrected for optical distortion (Section 2.4);
  - the center of the Sun's disk is determined as described in Section 2.3.2;
  - the center coordinates are corrected for the change of solar declination relative to the first image of the series (Equation (4));
  - the center coordinates are corrected for the change of the declination caused by the refraction, relative to the first image of the series (Equation (6));
  - the correction implied by Equation (3) is applied to the center coordinates.
- The angle ΔP_o is determined by fitting the trajectory of the disk center of the series to a straight line (with the iterative approach described in Section 3). Figure 3 shows the straight line fitted to the centers of a solar image series taken with the heliograph at Gyula Observing Station.

5. General Approach to Determine ΔP_o

In the previous sections, we made no assumptions about the solar telescope, except that it is capable to take solar full-disk images. Thus, the measurement of ΔP_o at a given position of the Sun in the sky can be applied to any kind of solar full-disk telescopes. This is because the telescope does not move while taking the image series. Therefore, the center of the solar disk drifts along the geocentric east – west direction.

In the case of an ideal telescope the angle ΔP_o is constant or changes in a known way as the telescope follows the Sun on its diurnal motion. For example, for an equatorially mounted refracting telescope with an objective and a magnifying optics this angle is constant, but for an equatorially mounted refracting coudé telescope ΔP_o is a linear function of the hour angle and the declination (Győri, 1989). If the telescope is ideal, and we know how the solar image rotates in the image plane, then, in principle, a single ΔP_o measurement is sufficient. However, in practice there is no ideal telescope. Errors may result from its mount, mechanical structure, and optical system. Moreover, gravity may cause deformation in different parts of the telescope structure. As a result of these effects the angle ΔP_o will depend on the position of the Sun in the sky. Of course, it is impossible to measure ΔP_o at every

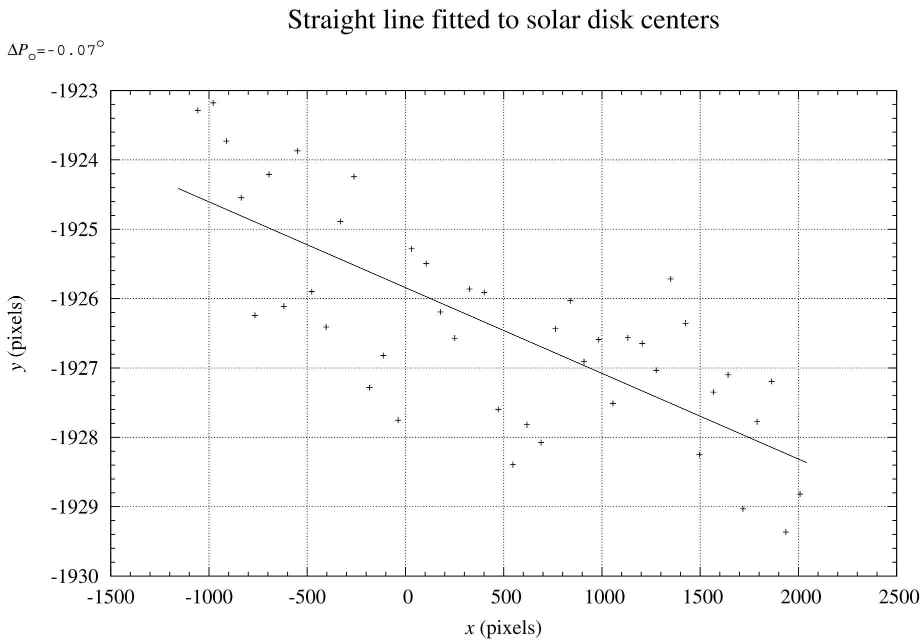

Figure 3 Straight line fitted to the centers of the solar disk in a drift series. The origin of the coordinate system is at the upper-left corner of the CCD.

position of the Sun. Therefore, we must somehow interpolate the unknown ΔP_{o} values from the measured ones.

We can take two approaches: a simple empirical approach or an approach that make use of the telescope model.

5.1. Empirical Approach

To determine the angle ΔP_{o} for any position of the Sun in the sky, and to reduce the effect of random errors in the measurements, a simple interpolation scheme can be applied. Suppose that ΔP_{o} is measured at several hour angles and declinations of the Sun. Then, for a given position of the Sun (even if there is a measurement for this position) ΔP_{o} is determined as the average of ΔP_{o} from the *near-by* measurements. A measurement at position $(t_{\mathrm{m}}, \delta_{\mathrm{m}})$ is considered to be near the position (t, δ), if the inequalities $|t_{\mathrm{m}} - t| < \Delta t$ and $|\delta_{\mathrm{m}} - \delta| < \Delta\delta$ hold (t and δ mean hour angle and declination, respectively). The proper values for Δt and $\Delta\delta$ can be determined from the measurements and will depend on how the telescope deviates from an ideal one.

In the case of solar telescopes that introduce image rotation, it is necessary to subtract the proper rotation angles from the measurements before the interpolation, and to add the proper rotation angles to the interpolated values after the interpolation.

Standard methods for interpolating a function on a regular grid (see, *e.g.*, Press *et al.*, 1986) can be used. In our case, however, measurements on a regular grid in the (t, δ)-plane are difficult to obtain. Moreover, the measured points are prone to errors.

5.2. Telescope Model

Using the principles and equations given in Győri (1989) we can follow the north direction of the Sun through the telescope. Thus we can construct a function that describes how the geocentric north direction of the solar image varies according to various telescope parameters. By fitting this function to the measured data, the parameter set in question may be determined. With these parameters, the orientation of the solar image can be computed from the constructed function for any position of the Sun in the sky.

6. Closed Formula for ΔP_{o}: The Common Solar Telescope

If a closed formula for ΔP_{o} can be derived for a telescope from the general scheme described in Section 5.2, then the parameters characterizing the telescope can be determined by fitting this formula to the measured ΔP_{o} values. Consequently, the value of ΔP_{o} can be calculated from the formula by using these parameters. This method is generally more accurate than the two general methods described in Section 5.

6.1. Deriving ΔP_{o} for the Common Solar Telescope

The common solar telescope is a refracting telescope with an objective and a magnifying optics. It has three axes, *i.e.*, the polar, the declination, and the optical. The optical axis of the objective and the magnifier coincide. Its mount is equatorial. A special case is the German mounting where the telescope can have two positions, *i.e.*, the east position when the telescope is on the east side of the pier and the west position when it is on the west side of the pier.

In the ideal case, where the axes of the telescope are perpendicular to each other and the hour axis points exactly to the north pole, ΔP_{o} is constant. Otherwise, it depends on the hour angle and the declination of the Sun and can be described according to Győri (1989) by

$$ {}^{\mathrm{E}}_{\mathrm{W}}\Delta P_{\mathrm{o}} = \lambda \sin(\varphi - t) \sec\delta \mp (\omega \sec\delta - \varepsilon \tan\delta) + \alpha, \tag{12} $$

where λ denotes the polar distance of the telescope's hour axis, φ is the hour angle of the telescope's hour axis, t is the hour angle of the Sun, δ is the declination of the Sun, ω is the deviation of the angle between the hour and the declination axes of the telescope from 90° (it is positive if the angle between the two axes is larger than 90°), ε is the deviation of the angle between the declination and the optical axes of the telescope from 90° (it is positive if the angle between the two axes is larger than 90°), and α is the angle between the declination axis and the rows of CCD. Here α is a constant that specifies the orientation of the CCD relative to the telescope.

The upper index E and the minus sign in Equation (12) refer to the east position of the telescope, while the lower index W and the plus sign indicate the west position. Of course Equation (12) is valid not only for the German mounting but also for other types of the common solar telescope with equatorial mounting. In this case, whichever form of Equation (12) was chosen, only the sign of ω and ε will change accordingly.

Fitting Equation (12) to the measured ΔP_{o} values, the parameters that describe the construction and alignment errors can be determined. Equation (12) then provides the orientation of the CCD detector at any time.

In the case of the German mounting, the process of parameter fitting can be broken down into two parts by adding and subtracting the ΔP_o values for the E- and W-positions of the telescope taken at nearly the same time, because from Equation (12) we have

$$\left({}^{\mathrm{E}}\Delta P_o + {}^{\mathrm{W}}\Delta P_o\right)/2 = \lambda \sin(\varphi - t) \sec\delta + \alpha, \tag{13}$$

$$\left({}^{\mathrm{E}}\Delta P_o - {}^{\mathrm{W}}\Delta P_o\right)/2 = -\omega \sec\delta + \varepsilon \tan\delta. \tag{14}$$

Equations (13) and (14) represent two curves (since $|\delta| < 24°$, we can approximate $\sec\delta \simeq 1$ with sufficient accuracy), while Equation (12) represents a two-parameter surface. Moreover, Equations (13) and (14) have separately fewer parameters to fit. Thus, it is simpler and more accurate to use Equation (13) and (14) than Equation (12).

The parameters λ and φ can also be determined in another way (Győri, 1990). Let us switch the telescope drive on and firmly clamp the telescope around its declination axis. If the telescope has an alignment error, the solar image is shifted along the geocentric north – south direction in the course of the day. The shift d, due to the misalignment of the telescope, can be written as

$$d = \lambda\left[\cos(\varphi - t) - \cos(\varphi - t_o)\right], \tag{15}$$

where t is the hour angle of the Sun and t_o denotes the hour angle at the beginning of the observation. The quantity d can be determined from a daily series of solar images as a function of the hour angle. By fitting the observed values of d to Equation (15), the parameters λ and φ can be obtained. We refer to this procedure as the daily shift method (DSM).

6.2. Measurements with the Gyula Heliograph

The Gyula heliograph is a common solar telescope with the German mounting. It has an Imacon Ixpress 96C CCD camera with 4080×4080 pixels.

Using the algorithm described in Section 4, we derived the values of ΔP_o for several positions of the Sun in the sky. For the two positions of the heliograph, Figures 4 and 5 show the measured ΔP_o as a function of solar hour angle. Since ΔP_o may also depend on the declination, a part of the scatter of the data may arise from this dependence. ΔP_o changes with solar hour angle, indicating that the polar alignment of the telescope is not perfect. A shift of about 0.05° between the E- and W-values of ΔP_o can also be observed. indicating that the angles between the telescope axes deviate from the right angle. Fitting the data to the curves given by Equations (13) and (14), the following values are obtained for the parameters: $\lambda = 0.278°$, $\varphi = -125.4°$, $\omega = 0.055°$, $\varepsilon = 0.034°$, and $\alpha = 0.205°$. As mentioned in Section 6.1, the alignment parameters (λ, φ) can be determined in another way, using DSM. Figure 6 shows the measured data and the fitted curve which were obtained by this method. The resulting alignment parameters are $\lambda = 0.172°$ and $\varphi = 140.8°$.

Comparing the results of the two methods, a discrepancy of 0.106° in λ and 15.4° in φ can be observed. This is a significant difference as can be seen in Figure 7, which shows the measured DSM data (+), the curve fitted to them (solid line), and the curve determined by the polar alignment parameters derived from the measurements of ΔP_o ($*$). Figure 7 also demonstrates the high sensitivity of DSM to the hour axis parameters. These results indicate that factors other than those mentioned hitherto may play a role, namely the flexure of the telescope (deformations in parts of the telescope caused by gravity).

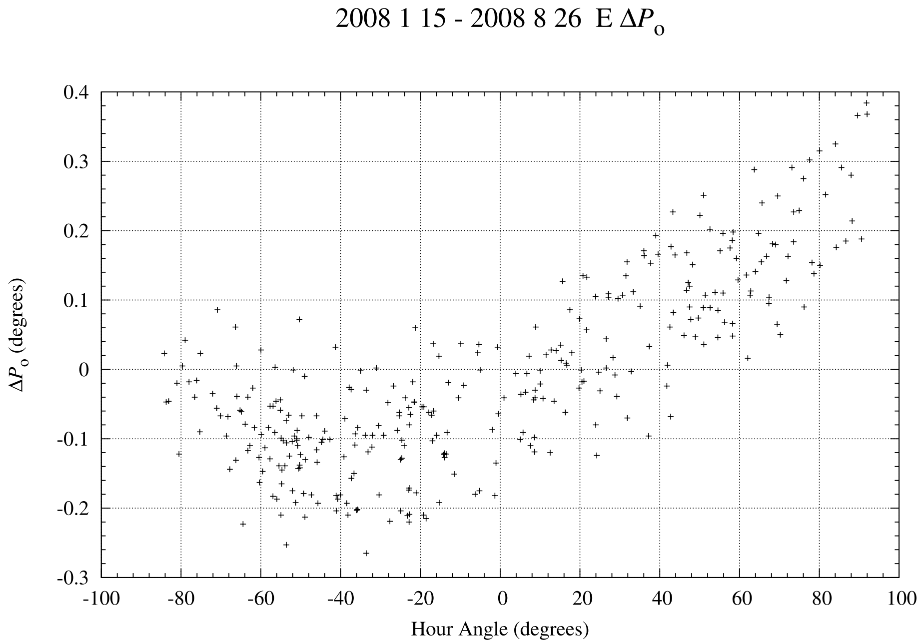

Figure 4 The measured ΔP_o as a function of solar hour angle for the E-position of the heliograph.

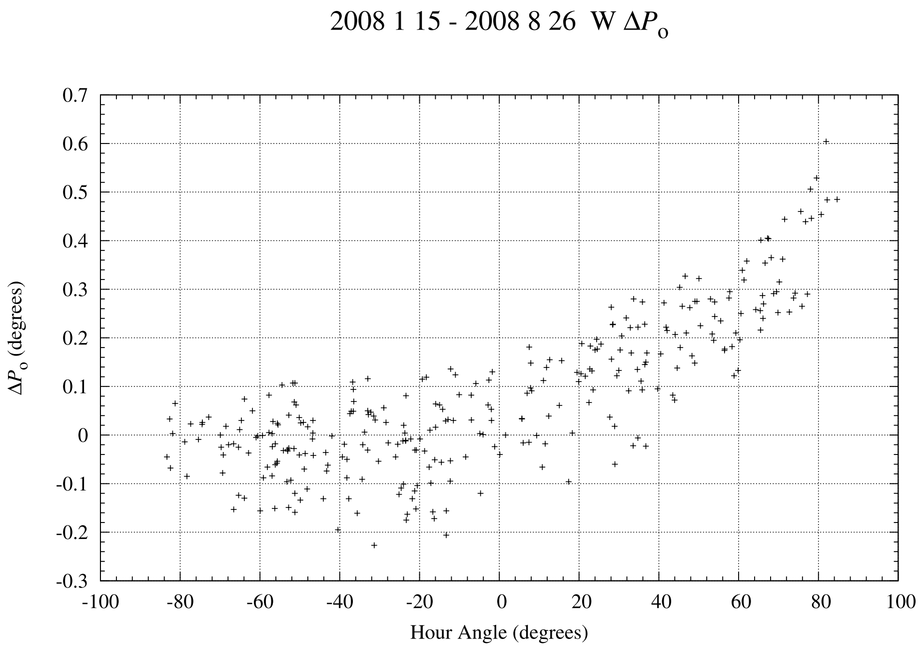

Figure 5 Same as Figure 4 but for the W-position of the heliograph.

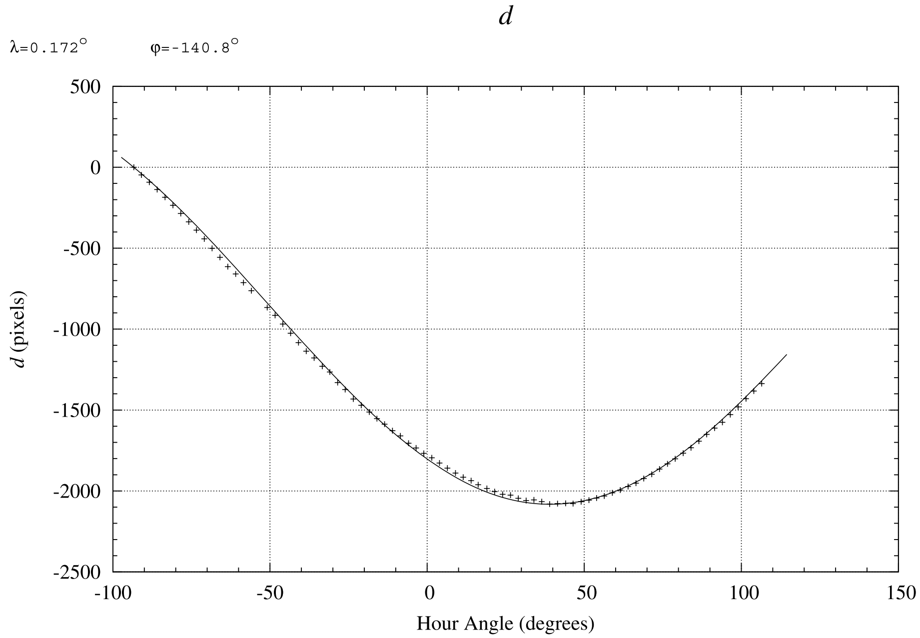

Figure 6 Displacement of the solar disk center along the geocentric north – south direction during the day, corrected for refraction and change in the declination, as a function of the solar hour angle. The solid line is the fitted curve.

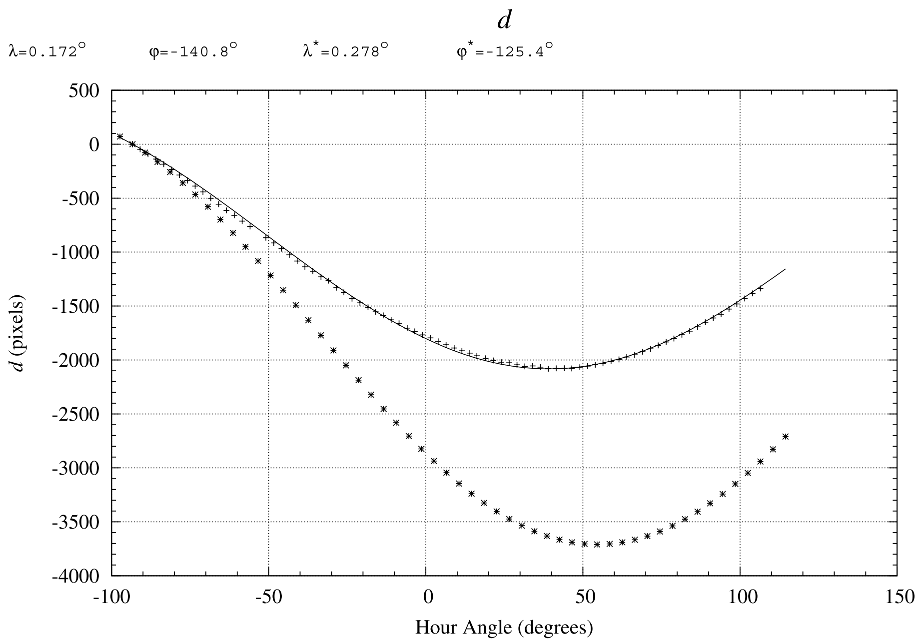

Figure 7 Measured DSM data (+), the fitted data (solid curve), and the curve determined by the polar alignment parameters derived from the ΔP_{O} measurements ($*$).

6.2.1. Telescope Flexure

The effect of the telescope flexure on the two types of the measurement is not the same: the measurement of ΔP_{o} is biased by those deformations of the telescope that rotate the CCD camera with respect to the geocentric north direction, while the DSM measurement is influenced by those that change the polar distance of the optical axis of the telescope. Therefore, if the telescope flexure is significant, we expect distinctly different values for λ and φ from the two methods.

Since the flexure of the telescope is a periodic function of the hour angle and the declination, it is appropriate to describe the change in ΔP_{o}, caused by the flexure, with a two-dimensional Fourier series. Therefore, if ΔP_{oF} denotes this change, we have, retaining only the zeroth and the first order terms,

$$\begin{aligned}\Delta P_{\mathrm{oF}} = {} & F_{\mathrm{o}} + F_{\mathrm{so}} \sin\delta + F_{\mathrm{co}} \cos\delta + F_{\mathrm{os}} \sin t + F_{\mathrm{oc}} \cos t \\ & + F_{\mathrm{sc}} \sin\delta \cos t + F_{\mathrm{ss}} \sin\delta \sin t + F_{\mathrm{cs}} \cos\delta \sin t \\ & + F_{\mathrm{cc}} \cos\delta \cos t,\end{aligned} \tag{16}$$

where the coefficients may depend on the position (E or W) of the telescope. Adding Equation (16) to Equation (12), we arrive at

$$\begin{aligned}\Delta P_{\mathrm{o}} = {} & F_{\mathrm{o}} + F_{\mathrm{so}} \sin\delta + F_{\mathrm{co}} \cos\delta + F_{\mathrm{os}} \sin t + F_{\mathrm{oc}} \cos t \\ & + F_{\mathrm{sc}} \sin\delta \cos t + F_{\mathrm{ss}} \sin\delta \sin t + F_{\mathrm{cs}} \cos\delta \sin t \\ & + F_{\mathrm{cc}} \cos\delta \cos t \\ & + \lambda \sin(\varphi - t) \sec(\delta) - (\omega \sec\delta - \varepsilon \tan\delta) + \alpha\end{aligned} \tag{17}$$

for the E-position of the telescope. Equation (17) describes how the angle ΔP_{o} varies in terms of the declination and hour angles of the telescope when the hour axis of the telescope does not point to the celestial pole, the axes of the telescope are not perpendicular to each other, and the flexure of different parts of the telescope cannot be neglected.

After some algebra, Equation (17) can be recast in the form

$$\begin{aligned}\Delta P_{\mathrm{o}} = {} & M_{\mathrm{s}} \sin t + F_{\mathrm{sc}} \sin\delta \cos t + F_{\mathrm{ss}} \sin\delta \sin t + M_{\mathrm{c}} \cos t \\ & + \varepsilon_{\mathrm{m}} \sin\delta + \alpha_{\mathrm{m}},\end{aligned} \tag{18}$$

where

$$M_{\mathrm{s}} = F_{\mathrm{os}} + F_{\mathrm{cs}} \cos\delta - \lambda \cos\varphi \sec\delta, \tag{19}$$

$$M_{\mathrm{c}} = F_{\mathrm{oc}} + F_{\mathrm{cs}} \cos\delta + \lambda \sin\varphi \sec\delta, \tag{20}$$

$$\varepsilon_{\mathrm{m}} = F_{\mathrm{so}} + \varepsilon \sec\delta, \tag{21}$$

$$\alpha_{\mathrm{m}} = F_{\mathrm{o}} + F_{\mathrm{co}} \cos\delta - \omega \sec\delta + \alpha. \tag{22}$$

Since $|\delta| < 24^\circ$ for the Sun and thus $\cos\delta \approx 1$ and $\sec\delta \approx 1$, the quantities M_{s}, M_{c}, ε_{m}, and α_{m} can be taken as constants.

We can similarly proceed for the W-position of the heliograph. Equations (18), (19), and (20) explain the disagreement between the hour-axis parameters obtained from ΔP_{o} and

Table 1 Parameters ΔP_{o} for the E- and W-positions of the heliograph with their standard errors

Position	M_{s} (°)	F_{sc} (°)	F_{ss} (°)	M_{c} (°)	ε_{m} (°)	α_{m} (°)
E	0.139	−0.188	−0.001	−0.139	0.205	0.088
	∓0.007	∓0.077	∓0.028	∓0.018	∓0.061	∓0.014
W	0.174	−0.203	0.091	−0.202	0.082	0.226
	∓0.007	∓0.092	∓0.032	∓0.022	∓0.072	∓0.017

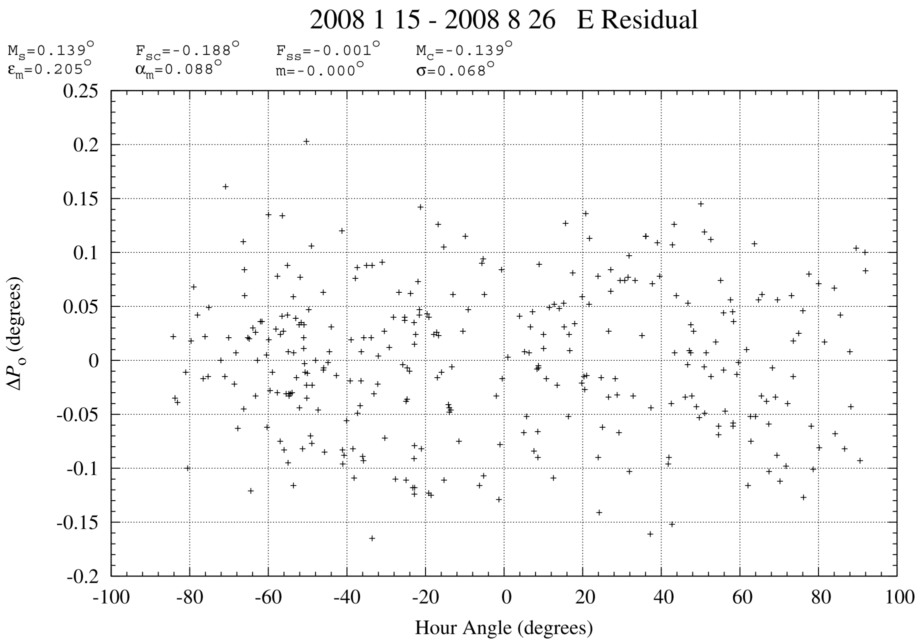

Figure 8 Residuals between the measured and calculated ΔP_{o} values for the E-position of the heliograph.

DSM measurements. As can be seen from these equations, if we have telescope flexure, only a mixture of the flexure, the hour axis, and the construction parameters can be determined by fitting Equation (18) to the measured ΔP_{o} data. The fitting of Equation (18) to the measured ΔP_{o} data at the E- and W-positions yields the parameters summarized in Table 1. Figures 8 and 9 depict the residuals between the measured and calculated ΔP_{o} values for the E- and W-positions of the heliograph, respectively.

6.3. Error Analysis

In connection with any method we can always pose the question: how accurate is it? As a first step to answer this question, let us determine the uncertainty of the slope of the line fitted to the solar disk centers in an image series, *i.e.*, the uncertainty of a single ΔP_{o} measurement.

The solar disk center is determined from the solar limb points. The refraction distorts the solar limb on many length scales (Győri, 1993). Before determining the center of the solar disk, a correction for the most systematic component of the refraction (known as the

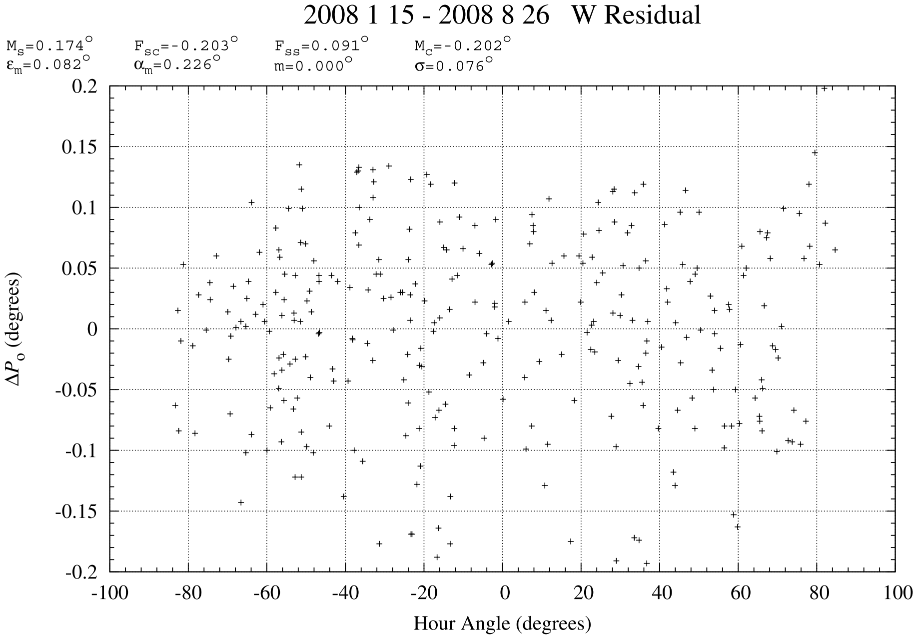

Figure 9 Same as Figure 8, but for the W-position of the heliograph.

astronomical refraction) is applied to the limb points (Section 2.3.2). But the refraction has a random component as well, which randomly moves the limb points, thus causing some uncertainty in the position of the solar disk center. Another effect is the vibration of the telescope (caused by the wind shake or the vibrations of the ground), which randomly moves the CCD detector relative to the solar image.

In our case, the CCD detector is aligned such that its rows are nearly parallel (within a few tenths of degrees) to the geocentric east – west direction. Therefore, the progression of the solar disk, during its transit across the CCD detector, is nearly parallel to the x-axis. Therefore, ΔP_o is not susceptible to the error caused by these effects in the x-coordinate of the solar disk center, because it occurs along the line to be fitted.

To estimate the error in the slope of the line fitted to the trajectory of the solar disk center, a Monte Carlo simulation was used. We assumed that the limb and the vibration errors in the y-coordinate of the solar disk center are normally distributed. The standard deviation of the normal distribution was chosen as 1.7 pixels. This value is the average standard deviation of the residuals between the fitted line and the measured disk center positions, derived from all image series. The synthetic solar disk center positions for the simulation were created by using the line with a slope of 0.1°. On this line 38 (the number of the images in a series) equidistantly spaced points were chosen, and their y-coordinates were randomly changed with numbers drawn from the normal distribution having the y-coordinate of the line for the point in question as the mean of the distribution and with the standard deviation of 1.7 pixels (0.88″ in the sky). The slope of the line was determined by fitting a line to the simulated center positions.

The above simulation was repeated several times (as many times as the number of the image series) to obtain the standard deviation σ_s of the slope of the lines fitted to the simulated image center positions. This value turned out to be 0.02°. The standard error obtained from the line fitting procedure applied to the measured disk center positions is 0.022° (this is the average over all the image series). The two values are in good agreement. Henceforth, we refer to the error in ΔP_o caused by the limb and vibration errors as the *slope error*.

On the basis of the above-mentioned facts we had expected the value 0.02° for the standard deviation σ_r of the residuals between the computed and the measured ΔP_o values. But this expectation was not fulfilled, because for σ_r we obtained the average value of 0.072° for the E- and W-positions, which is much larger than σ_s. The difference (hereafter called the *residual excess*) between the actual and the expected standard deviations of the residuals is 0.052°. This discrepancy can be clearly seen in Figures 8 and 9, which depict the residuals between the computed and the measured ΔP_o values. As a matter of fact, we must give up our expectation that the position (t, δ) of the telescope unambiguously determines the orientation of the CCD camera.

One obvious explanation would be some loose fastening somewhere in the telescope. We have carried out a thorough examination of the telescope, but we did not find any sign of such a problem.

Our solar telescope with its auxiliary equipments is a complex asymmetrical structure. In this structure, under the force of gravity, elastic deformation occurs. This deformation has a regular component determined by the position of the telescope (see Section 6.2.1), but it may have an erratic component, *e.g.*, random release of the stresses accumulated at certain locations inside the telescope causing shift or rotation in certain parts of the telescope. One candidate for the residual excess is this random release of stresses.

Another candidate is the clamping and fine adjustment system of the telescope. After taking an image series the declination and hour axes are unclamped, and the telescope is adjusted. Then, the axes are clamped, and finally the telescope is fine adjusted. The clamping and fine-adjusting system connects the telescope axes and its housing, exerting force on the axis when clamping or fine adjusting the telescope. This force may randomly change the angle between the declination and hour axes through the play of the declination axis in its housing. This change will result in a rotation of the CCD camera relative to the solar image (see Equation (12)).

The impacts of the slope error and the random effects that actually rotate the CCD detector relative to the solar image on the accuracy of the measured ΔP_o are quite different. Increasing the number of measurements (*i.e.*, the number of the image series) at the same telescope position will reduce the effect of the slope error to the measured ΔP_o (*i.e.*, the sample mean approaches the true mean). The situation is quite different in the case of random effects that actually rotate the CCD detector relative to the solar image. In this case, even by increasing the number of the measurements the accuracy of the measured ΔP_o will not be improved. Therefore, the residual excess of 0.052° can be considered as the accuracy of the orientation of our CCD camera.

7. Conclusions

In this paper, we presented how the traditional drift method for determining the orientation of a solar image can be adopted and expanded to achieve a high level of alignment accuracy for solar images taken with a CCD camera.

We have given a detailed treatment of the factors that determine the orientation of the CCD camera. Based on the results presented in this paper, a computer program was written, which was used to align the CCD camera attached to the Gyula heliograph.

As a byproduct, a new method for determining the optical distortion of the solar telescope was proposed.

The high accuracy of a single measurement has made it possible to reveal a surprising finding for the Gyula heliograph, namely that the orientation of the solar image is not uniquely determined by the position of the telescope in the sky, *i.e.*, the solar image orientation itself (not only its measured value) is a random variable at a given telescope position. The statistical property of this random variable may greatly influence the final accuracy of the orientation of the solar image. The implications of these findings and the developed method in general are applicable to other telescopes as well.

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