Astrophys Space Sci (2008) 318: 79–86
DOI 10.1007/s10509-008-9902-8

ORIGINAL PAPER

Flux-rms relation in solar radio bursts

Lin Wang · Cheng Fang · Yu Ying Liu

Received: 3 June 2008 / Accepted: 31 August 2008 / Published online: 20 September 2008
© Springer Science+Business Media B.V. 2008

Abstract One recent discovery that provides a strong constraint on the mechanisms of astrophysical activities is the correlation between the flux and the root-mean-squared (rms) variability of X-ray emission. In this work we study the flux-rms relation of solar radio bursts. Four flares observed by the *Solar Radio Broadband Spectrometer* (SRBS) of China are analyzed. In these flares, fine structures (FSs) emerge at least in one frequency band of SRBS. We find that the flux-rms relation consists of two components. One relates to the non-FS emission and the other to the FS emission. The flux-rms relationship for the non-FS part of the radio bursts is clearly different from that for the FS part. The former shows a curve-like behavior, while the latter shows a dramatic variation. We propose a model to describe the flux-rms relation of the non-FS part. Our results imply that the non-FS part emission could be triggered by some multiplicative processes. On the contrary, multiplicative mechanisms should be excluded from the explanations of FSs in the radio bursts.

Keywords Sun · Flares · Radio Radiation

1 Introduction

Solar flares are thought to be consisted of numerous elementary flare bursts (EFBs) which are considered as the results of small-scale reconnections occurring in the coronal magnetic field. The observational signature of EFBs is probably the fast fluctuation called spikes in the light curves of hard X-ray (HXR) and microwave bursts, which are known to be emitted by the non-thermal electrons precipitating into the low corona and the chromosphere through bremsstrahlung and gyrosynchrotron radiation, respectively. Kiplinger et al. (1983) found hundreds of spikes with subsecond duration from a sample of about 3000 HXR solar flares observed by the HXR burst spectrometer on board the *Solar Maximum Mission* (SMM). The duration of the fastest spikes can be as short as 45 ms. Aschwanden et al. (1998) statistically analyzed the HXR and radio elementary temporal structures and their frequency distributions in solar flares, and found that the time profiles of the resolved elementary structures have a near-Gaussian shape while the frequency distributions can be fitted by power-law or exponential functions. Fine structures (FSs) superimposed on microwave bursts are also frequently observed in microwave and decimetric emissions (e.g., Benz et al. 1992; Fu et al. 1995; Kliem et al. 2000; Nakajima 2000; Jiřička et al. 2001; Fu et al. 2003). The typical features of the FSs are very short duration, narrow frequency bandwidth, high polarization degree and irregular sequence in frequency and time. The duration of the individual FS is comparable to the electron-ion collision time, ranging from several to hundreds of milliseconds. Both the duration and the number of FSs decrease with frequency increasing (Fu et al. 2003).

However, due to the limitation of the spatial resolution of observations, one can not observe directly the spatial structures of the EFBs and FSs. The mechanisms of the EFBs and FSs are still not well understood. One recent discovery that provides a strong constraint on the mechanisms of astrophysical activities is the correlation between the flux

---

L. Wang (✉) · C. Fang
Department of Astronomy, Nanjing University, Nanjing, 210093, China
e-mail: wanglatnju@yahoo.cn

Y.Y. Liu
National Astronomical Observatories, Chinese Academy of Sciences, Beijing, 100080, China

Table 1 Information of the selected flares

GOES			Duration of FSs[a]		
Date	Peak	Class	1.0–2.0 GHz	2.6–3.8 GHz	5.2–7.6 GHz
2002 Apr. 21	01:30:24	X1.5	no data	7 min	no FSs
2004 Nov. 30	06:56:00	C4.8	6 min	no bursts	no bursts
2005 Jul. 14	11:04:56	X1.2	11 min	3 min	no FSs
2005 Jul. 30	06:25:41	X1.3	10 min	34 sec	no FSs

[a] An approximation obtained from our FSs identification

and the root-mean-squared (rms) variability of X-ray emission. Specifically, it was found that the absolute amplitude of rms increases linearly with the mean flux. A linear flux-rms relation together with a log-normal flux distribution imply that the underlying physical process is multiplicative, rather than additive (such as shot-noise), or related to self-organized criticality, or a result from completely independent variations in many separate emitting regions (Uttley et al. 2005). The linear flux-rms relationship is observed for many quite different objects, such as active galactic nuclei, X-ray binary systems (e.g., Uttley and McHardy 2001; Edelson et al. 2002; Uttley 2004), neutron stars (Uttley 2004) and even in a gamma-ray burst. Recently, Zhang (2007) analyzed the X-ray variability in the solar X-ray light curves collected by the *Reuven Ramaty High Energy Solar Spectroscopic Imager* (RHESSI; Lin et al. 2002). The temporal resolution of the RHESSI X-ray data used in his work is 0.1 s. He also found a similar linear relationship of flux-rms over multi-scales. The author suggested that the basic X-ray emission elements are interconnected.

Motivated by these results and considering that both the hard X-ray and the microwave bursts are produced by non-thermal electrons accelerated in solar flares, we examine the flux-rms relation in microwave bursts observed by the *Solar Radio Broadband Spectrometer* (SRBS; Fu et al. 2004) of China. The high temporal resolution of the SRBS is conducive to the study of variability.

This paper is organized as follows. The observations and the data reduction are described in Sect. 2. Our results are given in Sect. 3. Discussions and concluding remarks are given in Sect. 4.

2 Observations and Data Reduction

The SRBS has five frequency bands: 0.7–1.5 GHz, 1.0–2.0 GHz, 2.6–3.8 GHz, 4.5–7.5 GHz, and 5.2–7.6 GHz. The observations are made routinely at all of these bands in flux and polarization with high temporal and frequency resolutions except for the 4.5–7.5 GHz band, which has only the flux observations. The observation flow has been described by Fu et al. (1995). We briefly describe it as follows. After the emission passes through the antenna system, it reaches a microwave switch which is installed to perform the selection of left and right hand circle polarization (LHCP and RHCP) components by turns. Then, the emission is divided from the whole frequency band to many narrow frequency channels by a spectrum analyzer. Finally, after a digitization process, the dynamic spectrum is recorded.

We select four flares with different importance in the period of 2002–2005. In our work, data acquired from three bands are analyzed. The temporal and frequency resolutions are 1.25 ms and 4 MHz for 1.0–2.0 GHz band, 8 ms and 10 MHz for 2.6–3.8 GHz band, 5 ms and 20 MHz for 5.2–7.6 GHz band, respectively. Table 1 gives a summary of the selected events.

The data calibration is performed according to the method presented by Yan et al. (2002). The authors pointed out that the error of the calibration for moderate cases is below 10%. After the calibration, the two components, e.g., LHCP and RHCP, are added together to obtain the flux density in the solar flux unit (sfu). In order to get the statistic samples, a time interval is defined beforehand. Note that it should be long enough to ensure the statistic convincing. Then, for the light curve of a given frequency channel, we calculate the average flux density, $\overline{Flux}$, and the rms for all the data points that fall in the time interval. Note the rms is defined as:

$$\mathrm{rms} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(Flux_i - \overline{Flux})^2}, \tag{1}$$

where N is the number of the data points in the given time interval. To confirm the validity of our analysis on the rms, we compare the contributions from the instrument and the quiet-Sun with that acquired in our work. According to Fu et al. (2004), the sensitivity of the instrument is about 2% of the solar background radiation. Thus, the noise from the instrument and the quiet-Sun are estimated to be about $1 \sim 5$ sfu for the frequency ranges involved in our work, according to the quiescent flux given by the *Solar-Geophysical Data* (SGD). Note that they come principally from the receiver. Hence we can infer that they are roughly steady during the bursts. On the other hand, the rms calculated from the radio bursts is about $20 \sim 50$ sfu or higher (see the plots below). Since the rms of the radio bursts is concerned in this work, we conclude that the contributions from the instrument and the quiet-Sun cannot affect the results much. Other

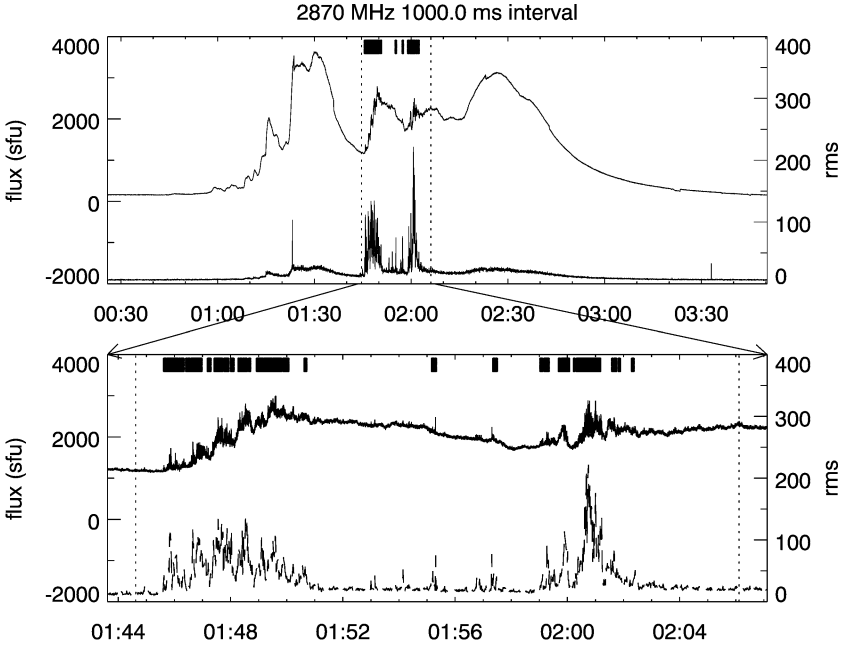

Fig. 1 An example showing the result of our FS identification method. *Upper panel*: the light curve (*upper part*, *left units*) of the 2002 April 21 flare and the corresponding rms (*lower part*, *right units*) for the given frequency channel. Two vertical lines show the period during which the FSs frequently appear. *Lower panel*: an enlarged plot of the period indicated in the upper panel. The blocks on the top of the two panels show the FSs-related data points identified by our method

contributions, such as the atmospheric and ionospheric fluctuations, are negligible, as is known for radio emission in gigahertz bands that is involved in this work.

It is believed that the emission mechanism of the FSs is remarkably different from that of the non-FS radio bursts. Hence it is important to distinguish the FS samples from the non-FS ones in our statistic study. To this end, we develop a simple method. First, we get the flux curve at a certain frequency channel with original temporal resolution. Then a segment of about 10 min duration before the flare onset is extracted and the standard deviation (δ) of the flux is calculated. Next, for a given time interval used in the flux-rms sample calculation, we search the data points having flux larger than the mean value in the given interval plus $4 \times \delta$.

It is noticed that there are some large noises that haphazard occur during the radio bursts. They appear as isolated data points and may meet the above criterion. To suppress the effect of these noises, a time window of 20 ms, which is comparable with the typical duration of FSs (Fu et al. 2004), is taken into account. That is to say, only the intervals that have successive data points, which meet the criterion and have a total duration no shorter than the time window (e.g., 16 points for 1.0–2.0 GHz band), are treated as FSs-related intervals.

Figure 1 gives an example to illustrate the validity of our FS identification method for the 2002 April 21 flare. From the upper panel, one can find that most of the FSs distribute in a short period. The rms for the FSs evidently shows a behavior different from that for the non-FS part. It suggests that the division of the two part is necessary. In the lower panel, the flux curve in the short period is enlarged. To indicate the points that are identified as FSs, we plot for each point a short line above the light curves. When the FS points are successive, the short lines will appear as black blocks. For the cases that there are one or some normal point(s) residing in the FS points, the blocks will interrupt. Thus, one can clearly find that the FSs are exactly identified. We experimentally checked this method for the data of the four flares in different frequency bands and found that it is good enough for our analysis. Thus we take this simple method as a good approach to search the FSs in the huge data set.

3 Results

3.1 Flux-rms relation for different time intervals

In order to find out the parametric effect of time intervals, we investigate first the variation of the flux-rms relation for different time intervals. Figure 2 gives examples showing the variation for the 2002 April 21 flare. Each flux-rms pair in a given frequency channel is calculated over a time interval of 200 ms, 500 ms, 1000 ms and 2000 ms respectively. Comparing the plots for different time intervals, we find again that the rms for the two components, i.e., those for non-FS part emission and those for FS emission, are different. The dispersion of the non-FS part component decreases as the increase of the time interval, which can be easily understood. We emphasize here that the curve-like behaviors of the flux-rms relationships for the non-FS part are similar. On the other hand, the relationship of the FS-part component looks unchanged. This fact confirms that the mechanisms of the two parts are intrinsically different.

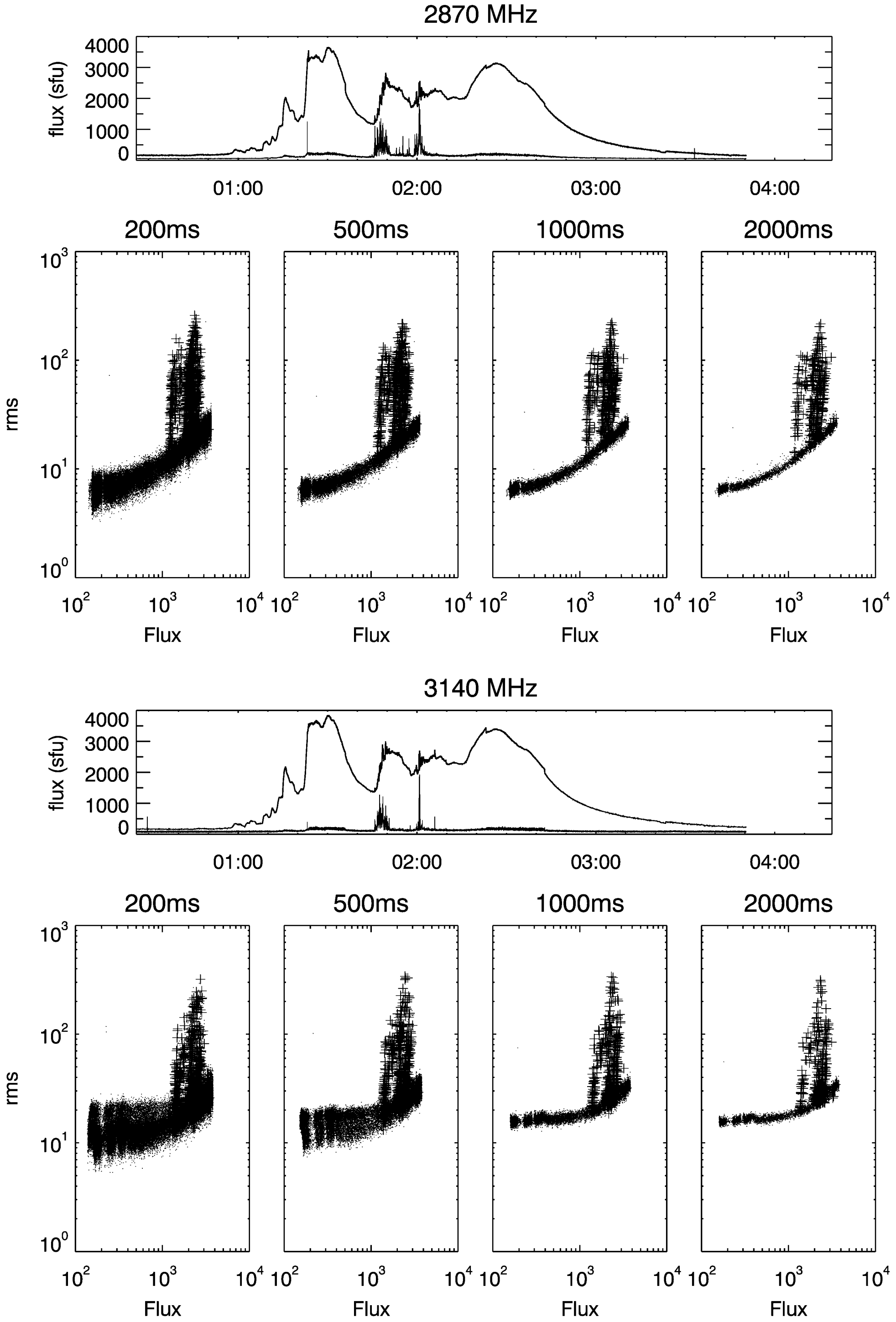

Fig. 2 Two examples showing the variation of the flux-rms relationship with different time intervals for the radio burst of the 2002 April 21 flare. *Upper panel*: the flux is plotted in the upper part (*left units*) and the rms in the lower part (*arbitrary units*). *Lower panel*: the flux-rms relationship. Four plots are given with the time interval of 200 ms, 500 ms, 1000 ms, and 2000 ms, respectively. The crosses represent the FS-part samples and the dots represent the non-FS ones

3.2 Flux-rms relation for the four flares

Figures 3–6 give the obtained flux-rms relations for the four flares at given frequencies. Each flux-rms pair is calculated over 1000 ms time interval. In the figures, the FS- and non-FS samples are plotted in crosses and dots, respectively.

It can be seen from the upper panels of the figures that the rms for the non-FS part of the radio bursts varies roughly with the flux. The samples of this component show a curve-like flux-rms relationship in the lower panel. However, the rms for the FS-part obviously deviates from this behavior. The dispersion of the rms for this part evidently larger than that for the non-FS part. No obvious flux-rms relationship can be found for the FS-part.

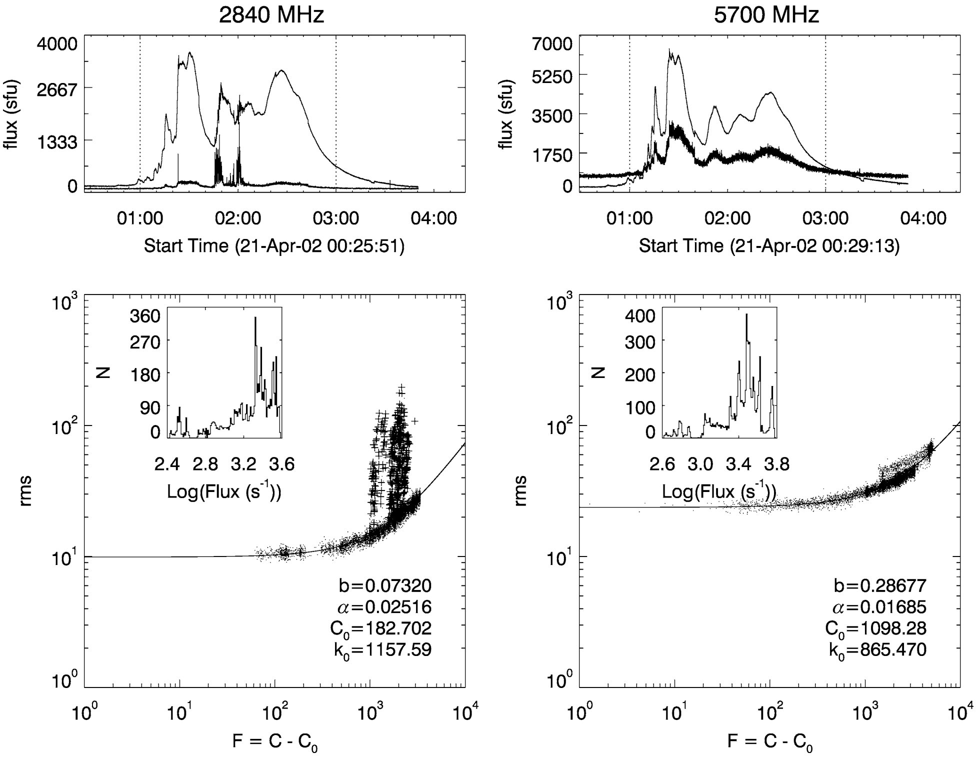

Fig. 3 Examples of the flux-rms relationship for the 2002 April 21 flare. The notation is similar to that in Fig. 2. The two vertical lines are the borderlines which constrain the samples in the modeling. The curves show the fitting result for the non-FS component. Note that the independent variable is F, rather than C used in Fig. 2. The inserts show the flux distributions

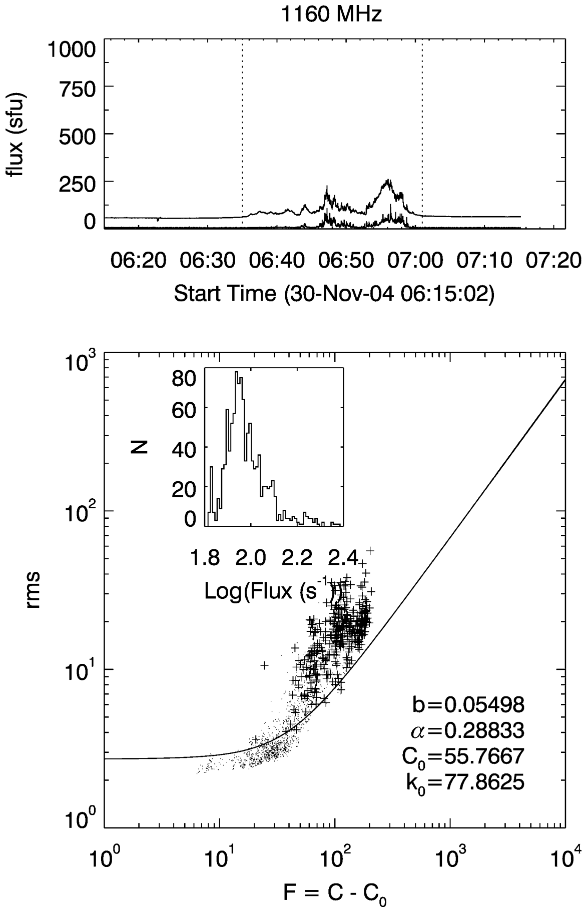

Fig. 4 Same as Fig. 3, but for the 2004 November 30 flare

3.3 Modeling of the non-FS part

Considering that the dispersion of the rms for the FS-part is very large within a small flux range, modeling of this component is unattainable. Hence, we only model the non-FS part.

Zhang (2007) pointed out that the flux-rms relationship for solar HXR bursts can be described by a model of

$$\mathrm{rms}^2 = (\alpha C)^2 + C, \tag{2}$$

where C is the average intensity within a time interval. For a more complicated case, he used a model of

$$\mathrm{rms}^2 = (\alpha F)^2 + C, \quad F = C - C_0, \tag{3}$$

where a constant parameter of C_0 is adopted. It may correspond to the background of the instrument in some cases (such as the GRB 940217) or represent an intrinsic component for some other cases (such as Cygnus X-1).

Comparing with the HXR emission, the radio bursts are different not only on the emission mechanism, but also on the level of the background emission. Therefore, for the case of solar radio bursts, we further generalize the model as follows:

$$\mathrm{rms}^2 = b[(\alpha F)^2 + C + k_0], \quad F = C - C_0, \tag{4}$$

The additional two parameters, b and k_0, make the model more general than (3).

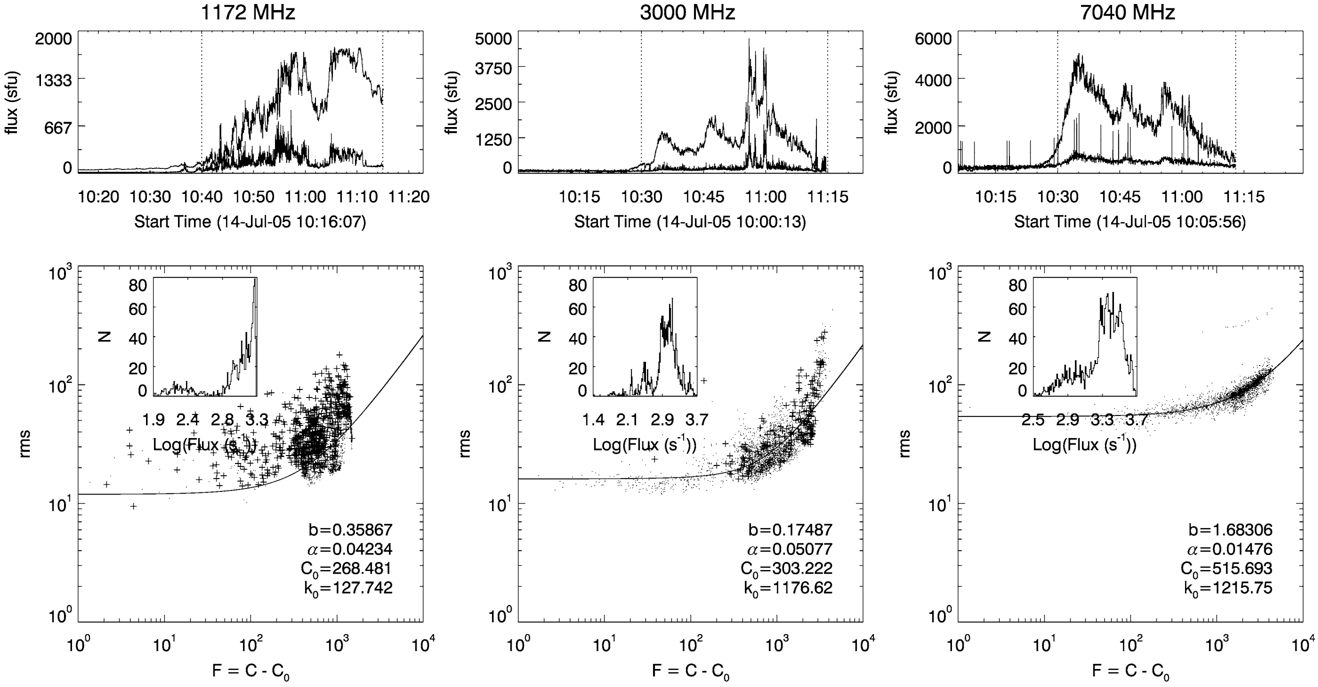

Fig. 5 Same as Fig. 3, but for the 2005 July 14 flare

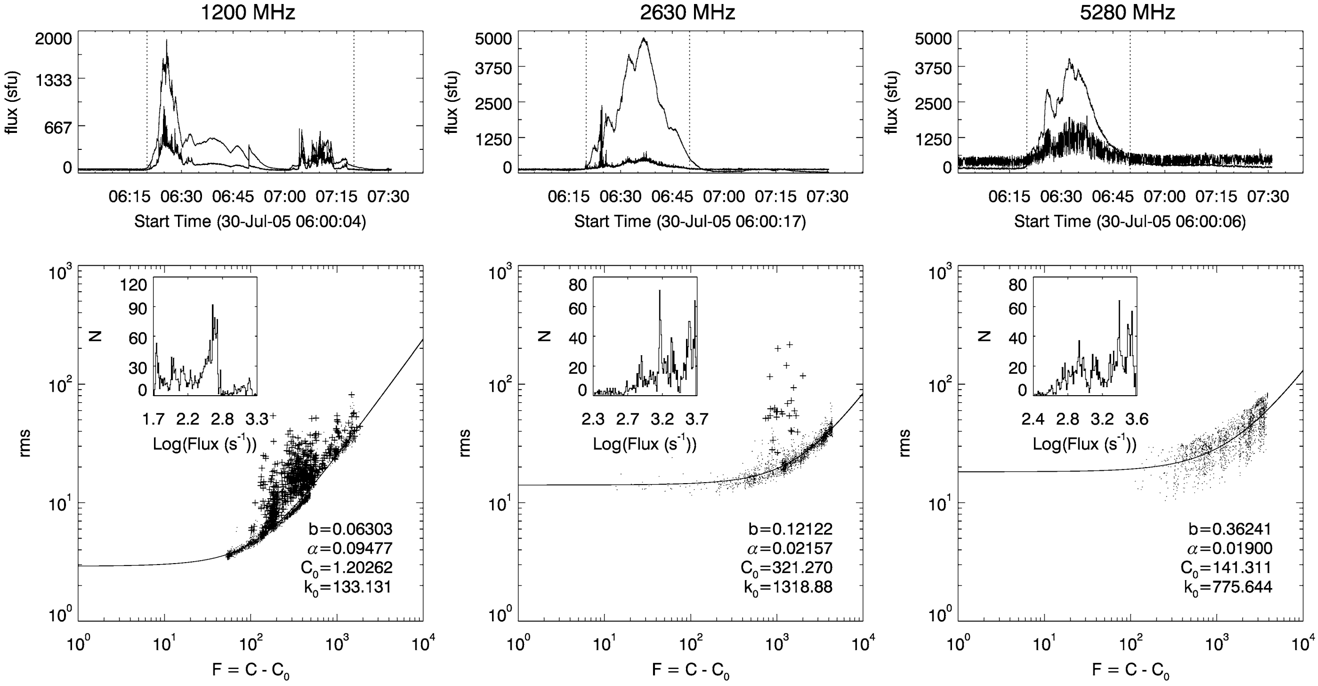

Fig. 6 Same as Fig. 3, but for the 2005 July 30 flare

The fitting of the radio flux-rms relationship is then done by using the model. We choose a time interval only covering the radio burst to suppress the effect of the quiescent emission. The time intervals are shown by the two vertical borderlines in the upper panels of Figs. 3–6. The results of the fitting are shown in the lower panels of these figures. Note that the independent variable is F (in sfu) rather than C. From the figures one can find that our model works well.

The success of the fitting ensures us to divide the non-FS component into two terms. The main term (i.e., $b(\alpha F)^2$, for the larger flux in the radio bursts) is linearly proportional to the flux. The other one (i.e., bC, or $b(C + k_0)$, for the smaller flux) is a Poisson fluctuation term, which is proportional to the square of the flux. Moreover, the inserts in the figures show the distribution of the flux. We find that almost all distributions can be approximated by one or several log-normal components, i.e. the logarithm of the flux follows normal distributions.

It is interesting to notice that the flux-rms relation mentioned above is suitable for the flares with different importance. As is shown in Fig. 4, the behavior of the flux-rms relation for the small flare of 2004 November 30 (Class C4.8) keeps the same as that for the X-class flares. It means that the flux-rms relation is an intrinsic property of the solar radio bursts.

4 Discussion and concluding remarks

By using solar radio observations with high temporal and frequency resolutions, we have analyzed the flux-rms relation for four flares. It is shown that the flux-rms relation consists of two components. The non-FS-related component exhibits a curve-like relation and can be modeled. According to our model, the rms is made up of two terms. For larger flux during bursts, the majority contribution comes from a linear component proportional to the flux. For small flux, the Poisson fluctuation term is non-negligible, which is proportional to the square of the flux. On the other hand, the flux-rms relation for the FS-part of the radio bursts is completely different. It can not be described through a simple model.

The linear component is crucial in the inference of the constraint on the physical conditions of solar bursts. However, before the inference, one must rule out the linear effect of the well-known time-bandwidth product in radio observations. The effect suggest that fluctuations of radio signals are proportional in some extent to fluxes divided by the square of the product of the integration time and the bandwidth. Thus it is important to correct this contribution before the data are presented. It should be taken into account the shape of the bandpass and any DC offsets in the signal chain. This seems unpractical at present. As a substituted approach, we compare the slope of the presented component with that of the linear effect. We find that the ratio of the two slopes are about 2–3, up to 6 in two cases. We take this as indirect evidence for the existence of linear relationship between the rms and flux. This may be taken as preliminary results at present. In the future, more samples of solar radio bursts including various types need to be analyzed, and the deduction of the instrument linear effect should be studied in detail beforehand.

As is indicated by Argollo de Menezes and Barabási (2004), if a dynamical system is externally driving, the rms should be linearly proportional to the flux; while for an internally driving system, the rms should be proportional to the square of the flux. For the cases that the acquired linear flux-rms relationship is reliable, one may infer that the non-FS part of radio bursts might be produced by some multiplicative processes driven externally. Moreover, Parker (1972, 1979) proposed that current-sheets may form at the boundaries of the braids, and finally induce topological dissipation, if the footpoints of a serious flux tubes are braided around one another. Following this scenario, one may infer, for the cases the linear relationship is reliable, that the bursts might be produced through numerous elementary ones, which are related with each others. In other words, the magnetic reconnections responsible for the non-thermal particle acceleration might somehow connected, rather than isolated, for the specialized cases.

Moreover, the log-normal distributions of flux showed in our results are inconsistent with the expectation of the avalanche model driven by self-organized criticality (Bak et al. 1988; Uttley et al. 2005). Hence the observations might suggest that the avalanche model is probably inappropriate to the explanation of the emission in solar flares.

The FS part in solar radio bursts has a more complicated flux-rms relationship. Thus, it can not be produced by some multiplicative processes. As is indicated by many observations, the characteristics of FSs are short duration, narrow frequency range, high bright temperature, and fast frequency drift, etc. Some authors suggested that the FSs are probably produced by electron cyclotron maser mechanism (e.g., Melrose and Dulk 1982) or by the plasma wave-wave interaction (e.g., Kuijpers et al. 1981; Vlahos et al. 1983). Anyhow, its physical mechanism is completely different from the non-FS ones. Moreover, our results indicate that any mechanism to explain the FSs should not be multiplicative. This provides a constraint on the emission mechanism of the FSs.

Acknowledgements We would like to thank the staff of the radio group in the National Astronomical Observatories of Chinese Academy of Sciences, who kindly provide the data and valuable discussions. This work was supported by the National Natural Science Foundation of China (NSFC) under the grant numbers 10221001, 10333040, 10403003, 10610099 and 10673004, as well as the grant from the 973 project 2006CB806302.

References

Argollo de Menezes, M., Barabási, A.L.: Phys. Rev. Lett. **93**(6), 068701 (2004)
Aschwanden, M.J., Dennis, B.R., Benz, A.O.: Astrophys. J. **497**, 972 (1998)
Bak, P., Tang, C., Wiesenfeld, K.: Phys. Rev. A **38**, 364 (1988)
Benz, A.O., Su, H., Magun, A., Stehling, W.: Astron. Astrophys. Suppl. Ser. **93**, 539 (1992)
Edelson, R., Turner, T.J., Pounds, K., Vaughan, S., Markowitz, A., Marshall, H., Dobbie, P., Warwick, R.: Astrophys. J. **568**, 610 (2002)

Fu, Q., Ji, H., Qin, Z., Xu, Z., Xia, Z., Wu, H., Liu, Y., Yan, Y., Huang, G., Chen, Z., Jin, Z., Yao, Q., Cheng, C., Xu, F., Wang, M., Pei, L., Chen, S., Yang, G., Tan, C., Shi, S.: Sol. Phys. **222**, 167 (2004)

Fu, Q., Qin, Z., Ji, H., Pei, L.: Sol. Phys. **160**, 97 (1995)

Fu, Q., Yan, Y., Liu, Y., Wang, M.: In: Henoux, J.C., Fang, C., Vilmer, N. (eds.) Understanding Active Phenomena, p. 55. World Publ., Beijing (2003)

Jiřička, K., Karlický, M., Mészárosová, H., Snížek, V.: Astron. Astrophys. **375**, 243 (2001)

Kiplinger, A.L., Dennis, B.R., Frost, K.J., Orwig, L.E., Emslie, A.G.: Astrophys. J. Lett. **265**, L99 (1983)

Kliem, B., Karlický, M., Benz, A.O.: Astron. Astrophys. **360**, 715 (2000)

Kuijpers, J., van der Post, P., Slottje, C.: Astron. Astrophys. **103**, 331 (1981)

Lin, R.P., Dennis, B.R., Hurford, G.J., Smith, D.M., Zehnder, A., Harvey, P.R., Curtis, D.W., Pankow, D., Turin, P., Bester, M., Csillaghy, A., Lewis, M., Madden, N., van Beek, H.F., Appleby, M., Raudorf, T., McTiernan, J., Ramaty, R., Schmahl, E., Schwartz, R., Krucker, S., Abiad, R., Quinn, T., Berg, P., Hashii, M., Sterling, R., Jackson, R., Pratt, R., Campbell, R.D., Malone, D., Landis, D., Barrington-Leigh, C.P., Slassi-Sennou, S., Cork, C., Clark, D., Amato, D., Orwig, L., Boyle, R., Banks, I.S., Shirey, K., Tolbert, A.K., Zarro, D., Snow, F., Thomsen, K., Henneck, R., McHedlishvili, A., Ming, P., Fivian, M., Jordan, J., Wanner, R., Crubb, J., Preble, J., Matranga, M., Benz, A., Hudson, H., Canfield, R.C., Holman, G.D., Crannell, C., Kosugi, T., Emslie, A.G., Vilmer, N., Brown, J.C., Johns-Krull, C., Aschwanden, M., Metcalf, T., Conway, A.: Sol. Phys. **210**, 3 (2002)

Melrose, D.B., Dulk, G.A.: Astrophys. J. **259**, 844 (1982)

Nakajima, H.: In: Ramaty, R., Mandzhavidze, N. (eds.) High Energy Sol. Phys. Workshop—Anticipating Hess! Astronomical Society of the Pacific Conference Series, vol. 206, p. 313. Astron. Soc. Pac., San Francisco (2000)

Parker, E.N.: Astrophys. J. **174**, 499 (1972)

Parker, E.N.: Cosmical magnetic fields: Their origin and their activity. Clarendon Press, New York (1979), 858 p., Oxford University Press, Oxford

Uttley, P.: Mon. Not. R. Astron. Soc. **347**, L61 (2004)

Uttley, P., McHardy, I.M.: Mon. Not. R. Astron. Soc. **323**, L26 (2001)

Uttley, P., McHardy, I.M., Vaughan, S.: Mon. Not. R. Astron. Soc. **359**, 345 (2005)

Vlahos, L., Sharma, R.R., Papadopoulos, K.: Astrophys. J. **275**, 374 (1983)

Yan, Y., Tan, C., Xu, L., Ji, H., Fu, Q., Song, G.: Sci. China Supp. **45**, 89 (2002)

Zhang, S.N.: Highlights of Astronomy **14**, 41 (2007)